THE TESTING OF

INSTRUMENT TRANSFORMERS

By

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PREFACE.

Although the testing of instrument transformers has formed the subject of a considerable number of papers published in the technical press during the past twenty years, the information contained in these papers has not, so far as the author is aware, been collected together in a form in which it is easily available for reference by those engineers who are interested in the operation and calibration of instrument transformers.

In 1920 the writer, at the suggestion of Professor T. Mather, F.R.S., undertook the preparation of a volume* dealing with the measurement of inductance and capacitance by alternating current bridge methods and, in examining the literature of the subject, accumulated at the same time some data on the closely related problem of testing instrument transformers for ratio and phase-angle. In 1923 Professor G.W.O.Howe suggested that the complete examination of the subject of instrument transformer testing, a correlation of published methods and a critical review of their utility would be of considerable practical value to engineers. This task the writer undertook to do, with the result embodied in the following pages.

The/

*"Alternating current bridge methods for the measurement of inductance, capacitance, and effective resistance at low and telephonic frequencies," Sir Isaac Pitman and Sons, 1923.

The subject is examined under four principal headings. Part I deals with general introductory matters and a discussion of the apparatus necessary for making tests in the laboratory or on site. Methods for testing current transformers are dealt with in Part II, while Part III is concerned with the methods used for calibration of voltage transformers. In Part IV the various methods are critically reviewed in order to determine those best suited to meet different practical conditions. Full references to technical literature are given and it is believed that no paper of importance has been omitted.

Finally, in preparing the numerous diagrams with which the text is illustrated, the author has had the invaluable assistance of Dr. M.G.Say, to whom thanks are accorded. The author wishes also to record his indebtedness to Professor Howe for his interest in the work, for encouragement in difficulties, and for provision of facilities without which the work could not have been done. The work was carried out in the Engineering Department of the University during the sessions 1923-24, 1924-25 and 1925-26.

Glasgow.
Feb., 1926.

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ABBREVIATED TITLES FOR REFERENCES.

Arch.f.Elekt...... Archiv für Elektrotechnik.

Atti dell' Assoc.Elett.Ital...... Atti dell' Associazione Elettrotecnica Italiana.

Beama...... Beama Journal (now World Power).

Bull.Bur.Stds...... Bulletin of the Bureau of Standards.

Bull.Schw.Elect.Ver...... Bulletin des Schweizerischen Elektrotechnischen Vereins.

Bull.Soc.Franc.des Elecns...... Bulletin de la Société Française des Electriciens.

Bull.Soc.Int.des Elecns...... Bulletin de la Société Internationale des Electriciens.

Dingler's Polytech.J...... Dingler's Polytechnicum Journal.

Elecn...... Electrician.

Elec.J...... Electric Journal.

Elec.Rev...... The Electrical Review.

Elec.World...... Electrical World.

Elekt.Zeits...... Elektrotechnische Zeitschrift.

Gen.Elec.Rev...... General Electric Review.

Journal I.E.E...... Journal of the Institution of Electrical Engineers.

Journal Frank.Inst...... Journal of the Franklin Institute.

Journal Sci.Insts...... Journal of Scientific Instruments.

L'Elettro...... L'Elettrotecnica.

Lum Elect...... La Lumière Electrique.

Phil.Mag...... The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science.

Proc.Amer.I.E.E...... Proceedings of the American Institute of Electrical Engineers.

Proc.Phys.Soc...... Proceedings of the Physical Society of London.

Rev.Gen.de l'El...... Revue Générale de l'Electricité.

Trans.Amer.I.E.E...... Transactions of the American Institute of Electrical Engineers.

Zeits.f.Inst...... Zeitschrift für Instrumentenkunde.

BIBLIOGRAPHY.

The following is a list of the books that have been consulted and in which the subject of instrument transformer testing is dealt with to some extent. These books also contain descriptive detail concerning the construction of instrument transformers, a matter not falling within the scope of the present work.

1. J.L.La Cour and O.S.Bragstad, Theory and calculation of electric currents; pp.328-335, gives theory of instrument transformers. 1913.

2. W.Jaeger, Elektrische Messtechnik; pp.121-124, theory of instrument transformers; pp.256-258, construction of standard transformers; pp.410-420, methods of testing. 1917.

3. F.W.Laws, Electrical Measurements; Chapter XII, pp.562-592, deals with construction, theory and testing of instrument transformers. 1917.

4. K.Edgcumbe, Industrial electrical measuring instruments; instrument transformers are examined at pp.312-340. Second edn. 1918.

5. A Dictionary of Applied Physics. Vol.II. Electricity; see the article Transformers, Instrument, by R.S.J.Spilsbury, pp.905-910. 1922.

6. C.V.Drysdale and A.C.Jolley, Electrical measuring instruments, part 2, pp.278-311 deals with construction, theory, and design of instrument transformers. 1924.

PART I.

GENERAL INTRODUCTION.

APPARATUS.

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CHAPTER I.

INTRODUCTORY.

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1. Preliminary.

In modern electrical engineering practice ammeters, voltmeters, and wattmeters, as well as relays and protective devices, used in alternating current circuits are invariably operated from the secondaries of instrument transformers. Moreover, in laboratory technique the use of transformers with precision instruments is very widespread, enabling a single instrument to be adapted to a number of scale ranges by merely changing the ratio of the transformer with which it is used. In either ~~case~~ instance the transformer becomes an integral part of the measuring unit and it is important to be able to determine its behaviour over a wide range of secondary load. This is especially necessary in the case of transformers used in connection with wattmeters and particularly with watthour meters, since the imperfections of the transformers may introduce considerable errors into the power or energy measurements, as the case may be.

The design and construction of instrument transformers to meet the conditions imposed by practice are, in themselves, sufficiently interesting topics, upon which sufficient exact information does not seem to be available in published papers. Indeed it seems to the writer that the design of such/

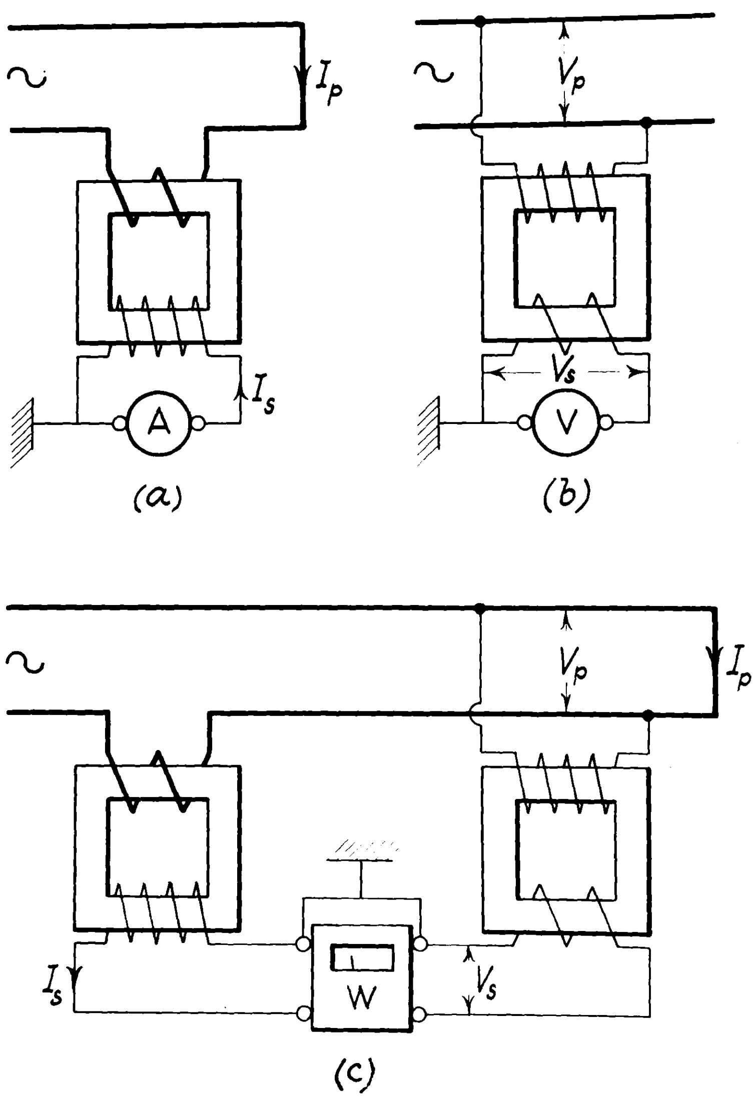

FIG. 1.

such transformers has very largely developed upon empirical lines suggested, no doubt, by tests that have demonstrated the imperfections of an earlier type of transformer and enabled the design to be modified so as to produce a better one. Suffice it to say that the modern instrument transformer, especially the voltage transformer, is a piece of apparatus of very high perfection and precision of action, so much so, indeed, that most refined methods are required to determine its small residual imperfections. In this volume the actual design and construction of transformers for use in a.c. measurements will not be examined but attention will be directed to a detailed consideration of the properties of such transformers, their imperfections, and the ways in which those imperfections may be practically determined. Questions relating to design will, however, be referred to in so far as they have a bearing upon the subject of instrument transformer testing.

2. The use of instrument transformers.

In-strument transformers are of two distinct types according as they are intended to deal with the main current or with the supply voltage. The current or series transformer, shown diagrammatically in Fig.1a, serves to transform a relatively large primary current I_p into a smaller current I_s which operates an ammeter A. The voltage, potential, or shunt/

shunt transformer, Fig 1b, similarly converts the high voltage V_p into a low voltage V_s operating the voltmeter V. The use of both types of transformer in conjunction with a wattmeter or a watthour meter is shown in Fig 1c. The instruments connected into the secondary circuits are invariably scaled in terms of the primary magnitudes.

Instrument transformers are used in all cases where heavy currents or high voltages are involved. By their use, instruments mounted upon the switchboard carry only a small current or are worked at a low voltage; hence no heavy current leads and no high voltage conductors are brought to the switchboard, with consequent cheapening of the wiring and greater safety for the operator. The insulation between the primary and secondary windings of the transformers serves to isolate all instruments from the high voltage of the main or primary circuit. The modern power station switchboard thus tends to become a low voltage instrument and control board, and as a further safeguard it is usual to earth the core and one secondary terminal of each instrument transformer. It will be realised, therefore, that instrument transformers must be insulated with the greatest care to withstand the high voltages that are impressed between primary and secondary windings. A current transformer, moreover, must be constructed in such a way that the windings will not become displaced under the action of the mechanical forces that act upon them when heavy/

heavy shortcircuit currents flow in the primary circuit. Thus, apart from any requirements as to accuracy of performance, instrument transformers must form a sound electrical and mechanical link between the main and the measuring circuits.

The use of instrument transformers is advantageous in another direction, since they can be chosen so as to render possible the standardisation of the secondary instruments. Modern practice tends to the adoption of a secondary current of 5 amperes in current transformers and a secondary voltage of 110 volts in voltage transformers, these values being laid down in the British Engineering Standards Association Specification No.81-1919 and in corresponding codes of foreign nations. The adoption of such a standard greatly cheapens the cost of manufacture of the secondary instruments and of the transformers themselves.

3. The rating of instrument transformers.

The behaviour of an instrument transformer depends, among other things, upon the amount and nature of the external secondary load or burden. The amount of the burden is usually stated as the load in volt-amperes which can be taken from the secondary winding without specified errors in transformation being exceeded. The B.E.S.A.Specification No.81, lays down for current transformers rated outputs of 15 and 40 voltamperes, and for voltage transformers 15, 50 and/

and 200 volt-amperes for adoption as standard values. Standard ratios of transformation are also specified. The importance of the amount of the burden, and particularly of its nature - to which the standard specification pays but little regard - upon the performance of instrument transformers will be reverted to in Parts II and III.

4. The Perfect Instrument Transformer.

A perfect instrument transformer is one in which the secondary quantity is an exact replica of the primary quantity to a reduced scale and in exact opposition of phase, independently of frequency, wave-form and secondary burden. Such a transformer does not exist in practice for a variety of reasons prominent among which are the following:-

(i) Some portion of the primary ampereturns is used to set up in the core of the transformer the magnetic flux requisite for the production of the secondary voltage.

(ii) The primary current contains a component accounting for loss of energy by hysteresis and eddy currents in the iron core.

(iii) The magnetic flux in the core does not link both primary and secondary windings equally, since there is inevitable magnetic leakage.

In consequence of these facts the secondary current in a current transformer is not equal to Primary turns/Secondary turns times the primary current; similarly in a voltage transformer the secondary voltage is not equal to Secondary turns/Primary turns times the primary voltage. Not only are the magnitudes of the transformed quantities in error but the secondary quantities are never in exact opposition of phase to the primary quantities. It is the measurement of the defects in ratio and in phase opposition that forms the subject of this volume.

By using a short magnetic circuit, preferably without joints, composed of highly permeable ferrous alloy with low magnetic losses, and by working at low magnetic flux densities it is possible to reduce the influence of the magnetising and loss components of the primary current upon the ratio and phase in an instrument transformer to a very small amount. The effects of leakage can be minimised by suitable interleaving and arrangement of the primary and secondary windings, so that on the whole the characteristics of the transformer approach the ideal but naturally never quite attain it. A more detailed analysis of the effects of magnetisation and leakage will be found in Parts II and III.

5. Definitions of ratio, ratio factor, and phase angle.

A current transformer is designed to give a definite nominal ratio between the primary and secondary currents. In a perfect transformer the ~~nominal~~ actual ratio would be constant under all conditions and exactly equal to the inverse ratio of primary to secondary turns; however, owing to the effects of ~~the~~ leakage the nominal current ratio K_{nc} always differs from/

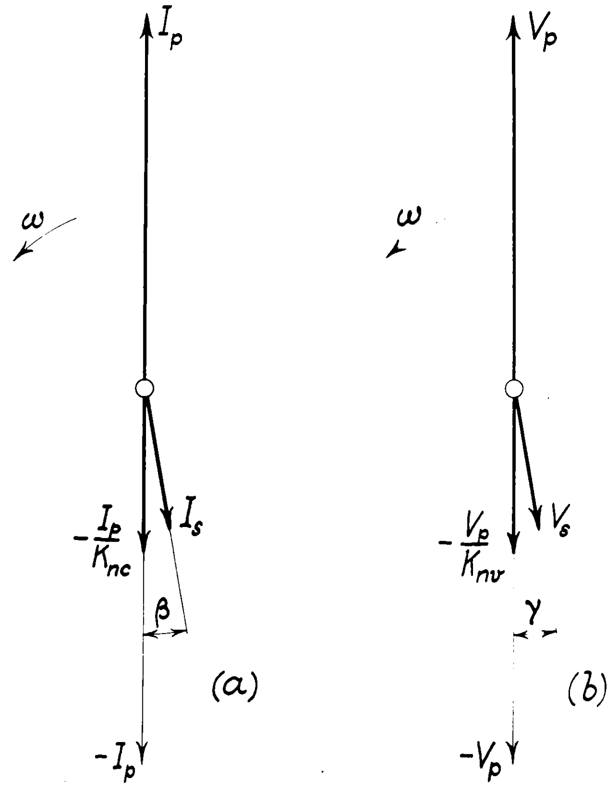
I_p
ω
I_s
$-\frac{I_p}{K_{nc}}$
β
(a)
$-I_p$
V_p
ω
V_s
$-\frac{V_p}{K_{nv}}$
γ
(b)
$-V_p$

FIG. 2.

differs from the ratio of secondary to primary turns. In consequence of the imperfections indicated in the preceding section, the actual ratio of primary to secondary current, see Fig.2a, differs from the nominal ratio. Thus in a transformer without ratio and phase error the secondary current would be I_p/K_{nc} and would be in direct opposition to I_p. In an actual transformer the true secondary current is I_s, usually less than I_p/K_{nc} and differing from opposition by a small angle β. Then

<u>Current Ratio</u> = $K_c = I_p/I_s$

The factor by which the nominal ratio must be multiplied to give the true ratio of current is the

<u>Current Ratio factor</u> = $F_c = K_c/K_{nc}$

A definition for β will be given in the following section.

In a voltage transformer, for similar reasons, the nominal ratio differs from the ratio of primary to secondary turns; moreover the actual ratio K_v itself differs from the nominal voltage ratio K_{nv}. The secondary voltage V_s is in opposition to the primary voltage V_p but for a small angle γ, the precise definition for which is given in the next section. Thus for a voltage transformer,

<u>Voltage ratio</u> = $K_v = V_p/V_s$

<u>Voltage ratio factor</u> = $F_v = K_v/K_{nv}$

The ratios K_c, K_v and the angles β, γ are dependent/

depandent upon frequency, the amount and nature of the secondary burden, the magnitude of the currents or voltages, and to a lesser extent on other factors such as wave-form. It follows that all these conditions must be exactly arranged in testing a transformer so that the values of ratio and phase-angle found by the test correspond with the conditions under which the transformer is used. It will be seen later that this is by no means easy to ensure.

In the case of a current transformer operating an ammeter, or a voltage transformer connected to a voltmeter, it is clear that correctness and constancy of ratio is all that is required, the phase displacement between the primary and secondary quantities being of no consequence. A voltage transformer works usually at practically constant primary voltage and if carefully constructed so that the magnetic circuit is good and the leakage small may be expected to be an instrument of precision. Tests show this to be the case, the error in ratio at any load being less than 1% and the phase-angle about 0.5° or less. A current transformer works under much less favourable conditions since it is expected to retain accuracy of ratio and phase over a wide range of primary current. The ratio error may lie between 1 and 2% and the phase angle at low loads may be 2 or 3°; better results can be obtained by careful design/

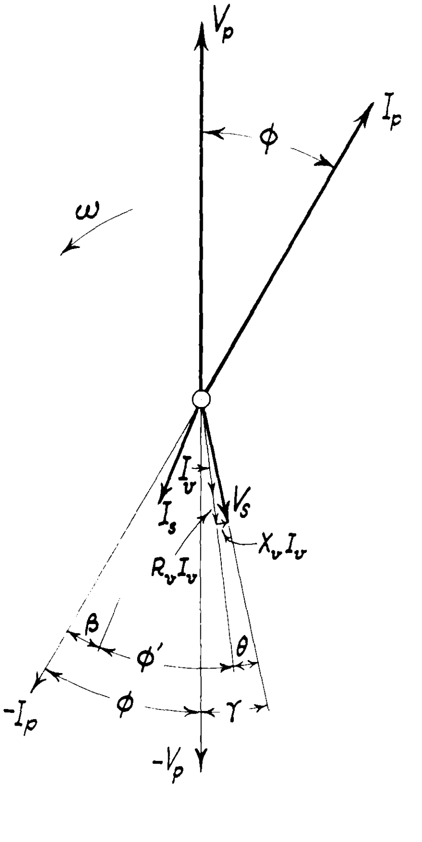
V_p
I_p
ϕ
ω
I_v
I_s
V_s
$X_v I_v$
$R_v I_v$
β
ϕ'
θ
ϕ
γ
$-I_p$
$-V_p$

FIG. 3.

design in transformers intended for use with precision laboratory instruments. It is natural, therefore, that greater attention has been paid to the improvement and testing of the current transformer than to the voltage transformer, as Parts II and III will show.*

6. Use of transformers with a wattmeter.

The indication of a dynamometer wattmeter is proportional to the product of the currents in its current and voltage coils and to the cosine of the phase difference between these currents. If a wattmeter be connected to a circuit via current and voltage transformers, errors will be introduced into its indications in consequence of the fact that the secondary quantities are not exact replicas of the primary quantities to reduced scale and are not in direct opposition thereto. Hence not only the ratio but also the phase-angle of instrument transformers used with a wattmeter are of prime importance, and even more so when used with a watthour meter.

Consider a wattmeter connected through transformers to a circuit, as in Fig 1c, to indicate the power therein. In Fig.3 let V_p and I_p be shown by vectors drawn from 0, ϕ being the phase angle between them so that

$$\text{True power} = V_p I_p \cos\phi.$$

The/

*For interesting discussions of ratio and phase definitions see M.G.Lloyd, "What is the ratio of a transformer?" Elec. World, vol. 52, pp.77-78, 1908; F.A.Kartak, "Testing of instrument transformers," Elec.World, vol. 75, pp.1368-1370,1920; A.Barbagelata,"Sulla prova dei trasformatori di misura",L'Elettrotecnica, vol.8,pp.165-175, 1921.

The voltage transformer applies a voltage $V_s = V_p/K_v$ to the volt coils of the wattmeter, V_s being shown in advance of $-V_p$ by an angle γ. Likewise the current applied to the wattmeter current circuit is $I_s = I_p/K_c$, shown leading by β on $-I_p$. The voltage V_s sends a current I_v through the volt coils which, in consequence of the reactance of these coils, will lag on V_s by an angle $\theta = \tan^{-1}(\omega L_v/R_v)$ as shown. The reading of the wattmeter as a dynamometer will be proportional to $I_v I_s \cos\phi'$, ϕ' being the angle between I_v and I_s; the reading in watts will be

$$\begin{aligned}\text{Wattmeter reading} &= R_v I_v I_s \cos\phi' \\ &= V_s \cos\theta. \times I_s \cos\phi' \\ &= V_s I_s \cos\theta. \times \cos(\phi-\beta+\gamma-\theta) \\ &= \frac{V_p I_p}{K_v K_c}\cos\theta \times \cos(\phi-\beta+\gamma-\theta)\end{aligned}$$

Hence,

$$\text{True Power} = K_v K_c \frac{\cos\phi}{\cos\theta.\cos(\phi-\theta-\beta+\gamma)} \times \text{Wattmeter reading}$$

If the transformers were free from phase error, the trigonometrical factor would become $\cos\phi/\cos\theta.\cos(\phi-\theta)$, which is the well-known correction factor for a wattmeter. From this it is apparent that a leading β and a leading γ have opposite influences upon the wattmeter; in fact, a leading angle β in a current transformer and

a/

a lagging angle γ in a voltage transformer each have an effect on the wattmeter equivalent to an increase in θ *.

The definitions for the phase-angles are thus conveniently taken as:-

The positive phase-angle β of a current transformer is the angle by which the secondary current leads on the reversed primary current.

The positive phase-angle γ of a voltage transformer is the angle by which the secondary voltage lags on the reversed primary voltage.

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* The subject is dealt with in detail in the following papers: L.W.Wild, Elecn., vol.56, pp.705-706, 1906; E.S.Harrar, Elec.World, vol.51, pp.1044-1046, 1908; K.A.Sterzel, Elekt. Zeits., vol.30, pp.489-491, 1909; W.Genkin, Lum.Elect., vol.8, 2nd series, pp.69-71, 1909; K.Edgcumbe, Elec.Rev., vol.67, pp.163-165, 1910; L.T.Robinson, Trans.Amer. I.E.E. vol.28,pp.1005-1039,1910; R.H.Chadwick, Elec World, vol.66, pp.13 1308-1310,1915;M.Rosenbaum, Elecn.,vol.74.,pp.626-630,1915; J.Goldstein,Bull.Schw.Elect.Ver.vol.11,pp.304-311,1920.

CHAPTER II.

CLASSIFICATION OF TESTING METHODS.

1. General.

Having briefly sketched the general properties of instrument transformers and defined their ratio and phase-angle, it is now necessary to review in a general way the methods that are used for the measurement of these important magnitudes. A fully detailed consideration of the methods applicable to the two types of transformers will be found in Parts II and III, the present object being to deal with broad principles.

Methods of testing instrument transformers are very numerous, but are divisible into two main groups, Indirect and Direct methods, which will be discussed in ensuing sections of this Chapter. The greater proportion of the methods are described in a large number of published papers scattered throughout the technical literature of Great Britain, the United States, Germany, France and Italy, but there are one or two* papers attempting some classification of the methods/

* R.S.J.Spilsbury, "Instrument Transformers," Beama, vol.6., pp.505-513, 1920; F.A.Kartak, "Testing of instrument transformers," Elec.World, vol.75, pp.1368-1370, 1920; A.Barbagelata, "Sulla prova di trasformatori di misura," L'Elettrotecnica, vol.8., pp.165-175,1921; and F.B.Silsbee, "Methods for testing current transformers," Trans.Amer. I.E.E., vol.43, pp.282-294, 1924.

methods which have been of great assistance in preparing the present work. Particular mention must be made of the valuable papers by Barbagelata and by Silsbee, where both laboratory and works methods are carefully considered.

2. The indirect method.

A most obvious method of finding the ratio and phase-angle of an instrument transformer is to utilise the experience gained in finding the regulation of a power transformer. The theory of the instrument transformer is first investigated in order to determine the way in which the ratio and phase-angle depend upon the magnetising characteristics of the iron core, the reactance and resistance of the windings, the secondary load, *etc*.. Measurements are then made upon the transformer to find the various quantities entering into the equations deduced for ratio and phase-angle, these measurements corresponding with the open-circuit and short-circuit tests made on power transformers but with some refinement in experimental detail in consequence of the smaller magnitude of the quantities involved. With the aid of these test results the ratio and phase-angle can be computed. The method has been used by a number of workers, especially for tests on current transformers for very large primary currents, since no specially constructed apparatus is required and the whole test can be made with calibrated ammeters, voltmeters and wattmeters of ordinary types.

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3. Direct Methods.

In contradistinction to the indirect method stand the direct methods, in which the ratio and phase-angle are directly measured by adaptation of the customary processes used for finding the ratio of two currents or voltages and the phase displacement between them. Direct methods are of two main classes; Absolute or Independent, wherein the ratio and phase-angle are determined in terms of laboratory standards directly upon the given transformer; Relative or Comparative, wherein the characteristics of a given transformer are compared with those of a standard transformer, the latter having been tested by an absolute method. Both Absolute and Relative methods are again divisible into two classes, designated Deflectional methods and Null methods. In the former the required characteristics are computed from the readings of appropriate instruments. In the latter it is usual to arrange a compensating circuit containing some form of alternating current detector, suitable reactance and resistance adjustments being made to reduce the deflection of the detector to zero; the ratio and phase imperfections are thus balanced out by the auxiliary circuit, the balance setting of which enables the desired quantities to be calculated. Null methods resemble in many points of principle and technique the well-known a.c. bridge methods for reactance and effective resistance measurement.

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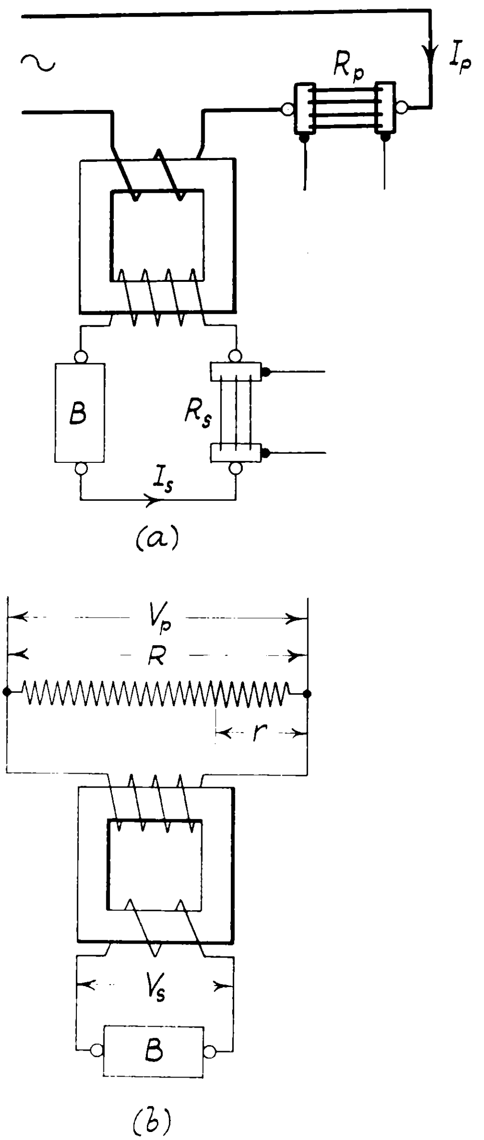

FIG. 4

The general principle of most direct absolute methods of testing current transformers is illustrated in Fig.4.a. A suitable four-terminal resistance R_p is put into the primary circuit and a similar resistance R_s in the secondary circuit; these resistances are chosen to give approximately equal drops of voltage and must be as non-reactive as possible. Moreover, since the inclusion of R_s in the secondary imposes a certain burden thereon, its value should be kept as small as possible; a combination of resistances and inductances B is inserted in the secondary so that in conjunction with R_s the burden with which the transformer will work in practice is imitated. Alternatively, B may be a suitable instrument, provided R_s is small enough in comparison therewith. The principle is then to compare the magnitudes of the voltage drops $R_p I_p$ and $R_s I_s$ and to find the phase-angle between them. This can be done in a variety of ways. In the deflectional methods the drops are compared in magnitude and phase by the use of one or more separately excited dynamometers or by the use of an electrometer; alternatively the resultant of $R_p I_p$ and $R_s I_s$ may be measured in a similar way. In the null methods this resultant voltage is compensated by an auxiliary voltage of adjustable magnitude and phase; adjustments are made until a suitable detector (vibration galvanometer, separately excited dynamometer, d.c. galvanometer and/

and synchronous commutator) shows balance to have been obtained. Other quite different procedures may be alternatively adopted; for example R_p and R_s may be replaced by suitable mutual inductors, the secondary voltages of which can be compared or their resultant balanced in a similar way. These and numerous other devices are fully explained in Chapters III and IV of Part II.

Relative methods of testing current transformers fall under two main principles. In the first the unknown and the standard transformers have their primaries excited in series; the behaviour of certain instruments when operated first from the secondary of the one transformer and next from the secondary of the other is observed. The instruments may be ammeters, dynamometers or still better, watt-hour meters, and the methods are essentially deflectional. The second principle is to excite the primaries in series and to set the secondaries in opposition, measurement being made of the resultant secondary magnitude. This may be done deflectionally by the use of a separately excited dynamometer or alternatively a null-method may be devised by the inclusion of an auxiliary circuit for compensation of this resultant and a suitable balance detector. These matters are considered in Chapters V and VI of Part II.

Absolute methods of testing voltage transformers are based on the principle illustrated by Fig.4b. A high resistance/

resistance R is put in parallel with the primary winding and the drop of voltage down a portion r of this is compared with the secondary voltage V_s . The comparison may be made deflectionally using a separately-excited dynamometer to measure the relative magnitudes and phase displacement of rV_p/R and V_s ; or the two voltages may be opposed through an auxiliary circuit provided with a means of correcting for the phase displacement between them,and balance secured by null indication of a suitable detector. Such methods are examined in Chapters I and II of Part III.

Relative methods of voltage transformer testing consist as a rule in supplying the primary windings in parallel from an a.c. source and comparing the secondary voltages of the standard and unknown transformers. The comparison may be made deflectionally, or by suitable compensating arrangements a null method may be devised. The matter is discussed in Chapters III and IV of Part III.

The object of the next Chapter is to review briefly the apparatus required in the testing of transformers in the laboratory or on site. The general plan is to deal first with the standards necessary for the purpose, then with sources of supply, and finally with instruments and detectors.

4. Order of precision in testing.

The order of precision demanded of a direct method of testing instrument transformers depends partly on the accuracy/

accuracy required in the transformer itself and partly on whether the tests are to be made in the laboratory with full experimental resources or on site by the use of portable instruments. For transformers that are to be used with precision instruments readable with the aid of a mirror below the pointer to 0.2% or less it is necessary to measure the ratio within 0.1% and the phase-angle to the nearest minute. A similar order of precision is requisite in tests on transformers used with watt-hour meters. Transformers operating switchboard instruments other than watt-hour meters are satisfactory if the ratio is known to 0.5%; those used with trip coils, phase meters, or protective devices may be tested to a much lower order of precision. Tests on transfommers of the highest class can only be made by sensitive laboratory methods. Transformers of less accuracy may be tested by methods of lower precision in the test room or on site using portable apparatus of no special design.

CHAPTER III.

APPARATUS.

1.Low Resistances.

It has been pointed out in the preceding Chapter that the primary and secondary currents of a current transformer are usually compared by passing each through a suitable low resistance and comparing the drops of voltage therein. Since the resistances are of a low value. invariably less than 1 ohm, they must necessarily be of the four-terminal construction so that the resistance may have a definite value. The construction design and theory of four-terminal resistances for use in d.c. circuits is well-known;* the problem is, however, much more difficult in the case of resistances for a.c. working owing to the influence of self inductance in the resistance and mutual inductance between it and its potential leads. In an ordinary four-terminal resistance or shunt, when an alternating current is passed into the current terminals a potential difference is set up between the potential terminals; if the shunt be free from inductive effects this voltage will be proportional to and/

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* G.F.C.Searle, "On resistances with current and potential terminals," Elecn., vol.66, pp.999-1002, 1029-1033, vol. 67, pp.12-14, 54-58, 1911. For extension of Searle's d.c. theory to a.c. working see F.Wenner, "The four terminal conductor and the Thomson bridge," Bull.Bur.Stds. vol.8, pp.559-609, 1913.

and in phase with the current. In general, however, without special arrangements of its components a shunt will possess some inductance, in consequence of which the current and voltage will be out of phase relatively by an angle determined by the ratio of the inductance to the resistance of the shunt, i.e., by its time-constant. Since the resistance is usually low it follows that before a shunt can be considered sufficiently nearly non-inductive for use in precise a.c. testing its inductance must be reduced to an exceedingly small value in comparison with its resistance. This is by no means easy to attain and a large amount of work has been done in trying to prepare non-reactive low resistances.

This non-reactive feature, about which more will be said later, is not the only one which a well-designed low resistance must possess. Those resistances used in the primary circuit of a current transformer when on test are frequently required to carry very large currents, up to a few thousand amperes, and must be designed to dissipate a considerable amount of power without undue temperature rise. (For example, a 0.002 ohm resistance for 1000 amperes absorbs 2 kw.) This necessitates a large cooling surface and such resistances may become very bulky. In addition it is generally necessary to provide artificial cooling, at any rate in the larger sizes; this may take the form of immersion of the resistance in a bath of well-stirred oil, of cooling the oil by a coil of pipe immersed therein carrying a flow of cold water, or alternatively the resistance/

resistance material may be directly water-cooled. The rise of temperature of the resistance can thus be minimised and by making the resistance of material such as manganin the change of resistance with temperature can be kept very slight. Permanence of value is also essential, and is secured by using well annealed manganin, hard soldering all joints and thoroughly ageing the resistance before use.

Reverting to the question of non-reactive construction, it is desirable not only that the resistance should have the lowest possible time-constant but also that the external magnetic field should be reduced to the minimum, so that interference with adjacent instruments is avoided. Two distinct principles of construction have been utilised. In the first method the resistance is made "closed", i.e. the main resistance material is arranged so that portions carrying current in opposite directions lie close to one another; hence the resulting magnetic field is made very small and the inductance slight. The potential leads are soldered to the resistance material near the current ~~electrodes~~ and are made short and so arranged as to be out of the influence of the small residual field of the shunt, as far as is possible. Resistances made in this way follow the well-known bifilar construction or some modification thereof. Orlich* in 1909 described/

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* E.Orlich, Zeits f. Inst., vol 29, p.241.1909; also H.Schering and E.Alberti; Arch. f. Elekt., vol 2 ~~6~~, pp.263-275, 1914, where dimensions of a set of bifilar resistances are given.

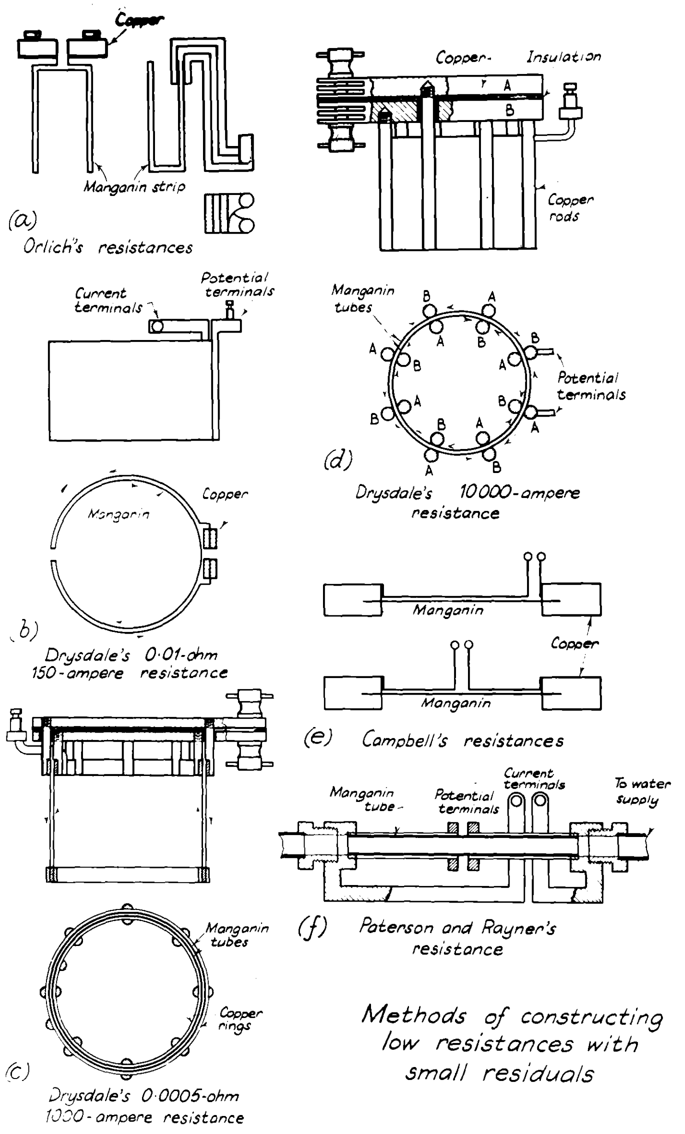
Copper
Manganin strip
(a)
Orlich's resistances
Current terminals
Potential terminals
Copper
Manganin
(b)
Drysdale's 0·01-ohm 150-ampere resistance
Manganin tubes
Copper rings
(c)
Drysdale's 0·0005-ohm 1000-ampere resistance
Copper
Insulation
A
B
Copper rods
Manganin tubes
Potential terminals
(d)
Drysdale's 10 000-ampere resistance
Manganin
Copper
Manganin
(e) Campbell's resistances
Manganin tube
Potential terminals
Current terminals
To water supply
(f) Paterson and Rayner's resistance
Methods of constructing low resistances with small residuals

FIG. 5.

described resistances, shown diagrammatically in Fig.5a composed of folded manganin strip with a mica separating layer between the two branches; potential terminals, not shown, are attached at the points where the manganin is soldered to the copper current leads. The measured time-constants are of the order of 2 to 5 microseconds. Sharp and Crawford,* following a bifilar design of resistance due to Drysdale, made resistances consisting of a number of units in parallel; a poor design of current terminal resulted in a time-constant of 17 microseconds. In 1916 Drysdale† described a number of designs for resistances, these being shown diagrammatically in Fig 5 b,c, and d. For the higher resistances the bifilar form is used. For lower resistances the material takes the form of concentric manganin tubes attached by short rods of copper to the current terminals. The latter consist of flat plates close to one another so as to be as non-inductive as possible. The tubular construction, in modified form, is applied in the lowest resistances in the way shown in Fig.5. d. The resistances are oil cooled, the oil being stirred mechanically and in the lower resistances cooled by water. A time-constant of about 0.75 microsecond is possible.

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*C.H.Sharp and W.W.Crawford, Trans.Amer. I.E.E., vol 29, pp.1517-1541, 1911.

†C.V.Drysdale, "New low resistance standards", Elecn., vol.77, pp.629-633, 1916.

In the second type of resistance the "open" type of construction is used, following a principle due to Lichtenstein* and applied to shunts by Campbell. The potential leads are in this case placed as near to the working resistance as possible, so that the self inductance of the resistance is nearly equal and opposite to the mutual inductance between the resistance and the potential leads. The p.d. at the potential terminals is then almost exactly in phase with the resistance drop in the shunt, the more nearly so as the shunt and potential leads are made to coincide. Campbell's straight resistances of this type are shown in Fig.5e. The principle has been employed in the resistances designed by Paterson and Rayner † for the N.P.L. shown in Fig.5f. The working resistance is a manganin tube soldered into copper end blocks; the potential leads are concentric copper tubes close to but insulated from the resistance tube. The resistance is used with the tube mounted vertically, a vigorous flow of cold water being maintained through it from bottom to top; by this means very efficient cooling is obtained and a current density in the manganin of 16000 amperes per square inch can be safely used. The resistances are designed for a normal volt-drop of/

*Lichtenstein, Dinglers Polytech. J. vol 321, p.100, 1906.
A.Campbell, "On compensation for self-inductance in shunt resistances", Elecn., vol.51.pp.1000-1001,1908.

† C.C.Paterson and E.H.Rayner, "Non inductive, watercooled standard resistances for precision alternating current measurements," Journal I.E.E., Vol.42, pp.455-470,1909.

of 2 volts, but if a temperature change of resistance of 2 parts in 10,000 can be tolerated much higher p.d. can be obtained. The following table gives the dimensions, maximum ratings and time-constants of the set of resistances:-

Resistance in Ohms.	Normal Current Amp.	Max. Cur-rent Amp.	Max. volt drop.	Outside dia. of tube. m.m.	Thickness of wall. m.m.	Length of tube c.m.	K.W. at max. cur-rent.	Time con-stant micro-sec.	Induct. ance in c.m.
0.04	50	115	4.6	6	0.25	35.5	0.53	0.16	6.5
0.02	100	260	5.2	10	0.30	40	1.35	0.27	5.4
0.01	200	450	4.5	15	0.40	39	2.00	0.34	3.4
0.002	1000	1300	2.6	30	1.00	48	3.40	1.85	3.7
0.001	2000	2500	2.5	40	1.50	42.5	6.25	3.00	3.0

The importance of a low time-constant can best be realised by a numerical example. Suppose a shunt of 0.001 ohm to have an inductance of 3×10^{-9} henry; then $L/R = 3 \times 10^{-9}/10^{-3} = 3$ microseconds. If used in a 50 cycle per second circuit the phase-angle would be nearly $\omega L/R = 100\pi \times 3 \times 10^{-6}$ radian = 0.000943 radian = 0.054° = 3.24 minutes. In measuring the phase-angle of a current transformer, lying as a rule between zero and about 120 minutes, this error in the phase of the voltage over the/

the shunt is by no means negligible.*

In many methods of current transformer testing an adjustable low resistance is often required. This is easily secured in practice by shunting a four-terminal resistance with a relatively high adjustable resistance box; additional shunting terminals are sometimes provided. Alternatively, low value slider resistances have been constructed† for use in such tests. These may consist of a low resistance wire or strip folded back on itself with insulation between the halves, the potential slider touching one wire or the edge of one half of the strip as the case may be.

2. High Resistances.

In the testing of voltage transformers a regular process is to join across the primary terminals a very high resistance or potential divider and to compare the drop of voltage down a fraction of this resistance with the secondary voltage of the transformer. Since transformers must be tested at the rated voltage it is essential for such potential divider resistances to be built to withstand the voltages usually encountered in practice, which may be many kilovolts. If such high resistances were wound with wire in the form/

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* For a very complete treatment of low resistances for a.c. work see F.B.Silsbee, "A study of the inductance of four terminal resistance standards," Bull.Bur.Stds., vol.13, pp.375-421.1917.

† See for example P.G.Agnew, Bull.Bur.Stds., vol.7, pp.423-474, 1911

form of a simple coil, the inductance of the winding and the electrostatic capacity from turn to turn would be considerable and the winding would not act in any way like a pure resistance when used in an a.c. circuit. The inductance can be reduced to a negligible amount by the well-known principle of bifilar winding; this, unfortunately, still leaves considerable electrostatic capacity between the turns of the coil. The inter-turn capacity effect, however, can be reduced as much as desired by using instead of a single bifilar-wound coil a number of such coils in series. The potential difference across each bifilar section of the winding is then for n coils only $1/n$ of the total p.d. and the capacity effect is reduced in proportion to $1/n^2$. Numerous alternative devices are used in practice* to effect the same result, e.g. bifilar sections wound on mica cards, Duddell-Mather resistance gauze, etc., and it is not difficult to build a very high resistance in which the total residual reactance, and therefore the time-constant is made very small. Such resistances, however, prove defective when used as potential dividers and for a very simple reason. A very high resistance naturally contains a considerable amount of wire and the bobbins or cards upon which the wire is wound present/

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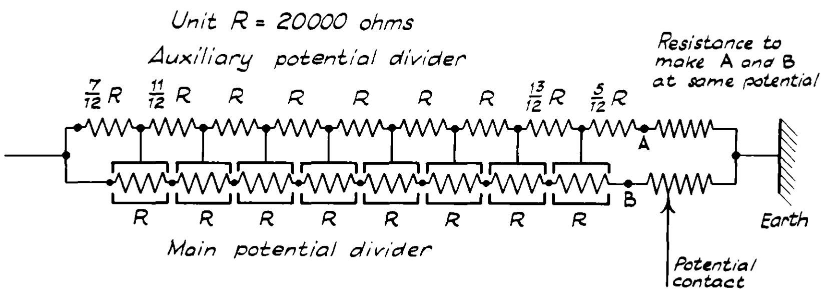
* For a full discussion of residuals in high resistances and methods for their reduction see B. Hague, Alternating Current Bridge Methods, pp.56-72, 1923.



Orlich and Schultze's screened potential divider

FIG. 6.

present a large area and have a relatively large electrostatic capacity with respect to surrounding earthed objects, this capacity being distributed over the length of the resistance. Now the fall of potential over any portion of the resistance is determined not merely by the resistance of the portion but also by its distributed earth capacity, exactly as in an a.c. transmission line. Hence the drops of potential down various fractions of the resistance are not by any means proportional to those fractions. This defect can be overcome by a device introduced by Orlich and Schultze* shown in its simplest form in Fig.6.. Each section of the potential divider is wound upon a separate porcelain tube encased in a separate metal screen, the potential of each screen being maintained at a value equal to the mean potential of the resistance section contained in it. This is done by connecting the screens to appropriate points upon a second high resistance joined in parallel with the first; ~~and~~ all resistances are wound in ~~bifilar~~ unifilar sections. By this means the earth capacity effects in the potential divider are reduced to a minimum and the voltage across its terminals is divided almost exactly in correct resistance proportions. At very high voltages, greater than 50 kilovolts, the earth capacity of the/

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* E.Orlich and H.Schultze, "Über einen Spannungsteiler für Hochspannungsmessungen," Arch.f.Elekt., vol.1.pp.1-15, 88-94, 232, 1913.

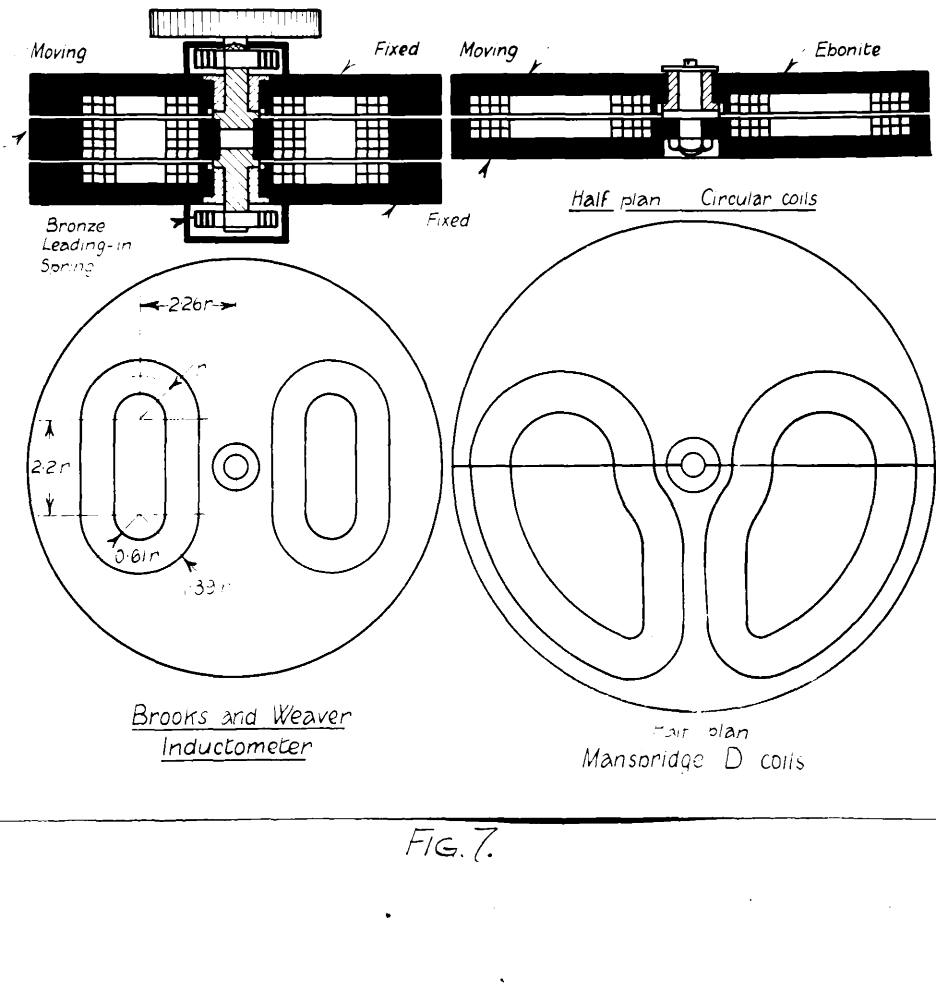
Moving
Fixed
Moving
Ebonite
Bronze
Leading-in
Fixed
Half plan Circular coils
2·26r
2·2r
0·61r
Brooks and Weaver
Inductometer
Mansbridge D coils

FIG. 7.

the auxiliary potential divider may become important; it is then necessary to screen its sections and to keep them at the correct potentials by the use of a second auxiliary potential divider.

3. Self and Mutual Inductors.

In some methods of testing instrument transformers variable self or mutual inductors are required, and it is essential that these should be of a type as free as possible from influence by external magnetic fields, i.e. of astatic construction. For use in low current circuits, not more than 5 amperes for example, the disc pattern of inductor is usually preferred. Three usual types* are shown diagrammatically in Fig.7.. Of these the Brooks and Weaver† pattern, specially designed originally for transformer testing, has the advantage of high time-constant, constancy of calibration during wear of the bearings and an almost uniform scale; it is probably the best instrument of its class. The four fixed coils are connected together and brought out to two terminals; the moving coils are in series between a second pair of terminals. Both sets are connected astatically. The dimension † is 36.8 m.m. and the overall diameter of the fixed discs is 35.5 cm. Each fixed/

* For a full discussion of the properties of these and other inductors, see B.Hague, loc.cit., pp.94-98, 1923.

† H.B.Brooks and F.C.Weaver, "A variable self and mutual inductor," Bull.Bur.Stds, vol.13,pp.569-580, 1917.

fixed coil is wound with 9 layers of 2 turns, ~~each~~ and each moving coil with 9 layers of 4 turns; the wire consists of 7 insulated copper strands each 0.8 m.m. diam.. The self inductance with fixed and moving systems in series varies between 125 and 1225 microhenrys; the mutual inductance between fixed and moving systems varies between -272 and +278 μH. The total resistance with the windings in series is 0.35 ohm and the maximum time-constant 3.4 milliseconds. This instrument may be taken as typical for the present purpose.

In some methods of testing current transformers mutual inductors are required with windings designed to carry large currents and which must be perfectly astatic. Fortescue has met these requirements by winding the inductors in toroidal form upon marble rings of circular cross section. If the windings are uniformly distributed round the ring perfect astaticism can be secured. The mutual inductance between primary and secondary is varied by varying the number of secondary turns in use. If the secondary be uniformly distributed, the primary may consist of a number of copper loops, equally spaced round the ring, which can be connected up in series, parallel, or series parallel groups, thus adapting the apparatus to a wide range of primary current. The apparatus is further described in Section 46 in Chapter IV of Part II.

4. Condensers.

The phase-angle of a transformer is sometimes compensated in testing methods by means of a suitable condenser inserted in an appropriate position in the test circuit. Such condensers must be accurate standards, of low loss, permanence of capacity, and freedom from frequency changes. These qualities are possessed by well-made mica condensers, which for ease of manipulation are best arranged in plug or switch operated decades. A total capacity of 1 μF. subdivided into decades of 1/10, 1/100, and 1/1000 μF. sections is convenient. The properties of such mica condensers have been fully discussed by various experimenters, the general conclusions being summarized by the author elsewhere.*

5. Transformers for use as standards.

In relative methods of testing instrument transformers a standard transformer, the characteristics of which are known and are preferably similar to those of the transformers under test, is required. All transformers, whether of current or voltage types, that are to be used as laboratory standards must be chosen for the permanence of their characteristics and for the small magnitude of their ratio and phase errors. This demands in particular that the iron circuit be composed of very good material with low iron loss and that the circuit be/

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*B.Hague, loc.cit., pp.102-104, 114-122, 1923.

be very carefully assembled, preferably without joints or at any rate with joints well interleaved and clamped. Moreover, the windings must be grouped upon the core so that magnetic leakage is reduced to a minimum. Only by attention to these constructional details can a satisfactory standard be prepared. Next only in importance to the proper construction of a standard transformer is its proper maintenance. It should be carefully used, always with its proper secondary burden connected; never in any circumstances should the core be magnetized by direct current or open circuited. From time to time the ratio and phase-angle should be checked by an absolute method. It is found in practice that with attention to these points a standard transformer is a very permanent and trustworthy piece of apparatus. No particular difficulty is encountered in establishing a standard voltage transformer, but the current transformer gives rise to a number of interesting matters that must now be briefly discussed.

Current transformers of normal types can, with care, be set up for use as standards. It is not uncommon to build transformers for laboratory use with multiple primaries which can be grouped in series, parallel or series-parallel to adapt the instrument for a variety of values of primary current. Experience shows this practice to be quite permissible since it is found that the ratio error and phase-angle of a well-made transformer of this type are almost independent/

independent of the grouping of the primary coils. Again, for testing on site it is not uncommon to use as a standard a ring type transformer, the iron ring being closely wound with a uniform secondary coil. The primary is improvised at the place of test by looping a current-carrying cable any desired number of times through the ring, thus making a transformer of any required ratio. In this case the ratio error and phase-angle depends very little upon the number and position of the primary turns through the ring, but the variation renders the instrument less suitable as a laboratory standard.

A number of suggestions have been made for the improvement of the current transformer, especially when used as a laboratory standard. One of the most obvious improvements is in the material of which the iron core is composed. Drysdale* has recently pointed out that by using such material as Permalloy or Mumetal, having a permeability about ten times that of good iron, a loss of about one sixth and a resistance about four times greater, the magnetising current could be reduced to 1/6 to 1/10 of the present value with iron, with corresponding reduction in the ratio and phase errors. Other investigators have designed standard current/

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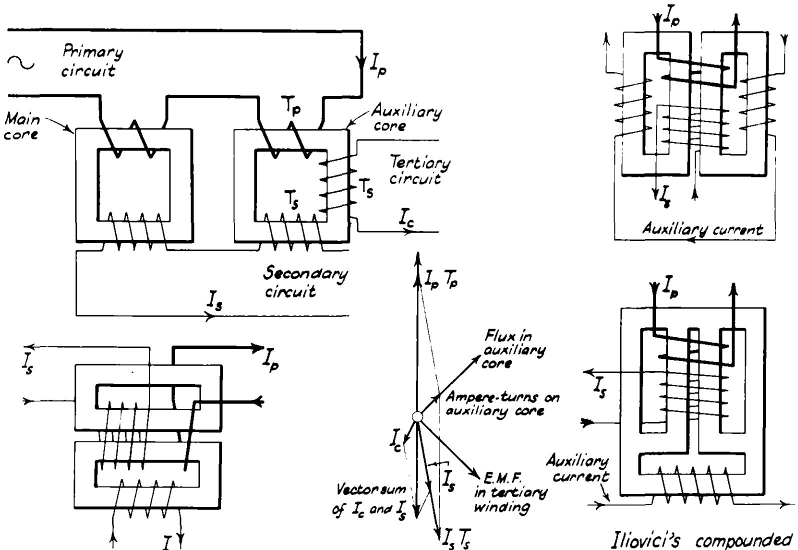
* C.V.Drysdale, "The application of high permeability alloys to current transformers," Journal.Sci.Insts. vol.3, p.58, 1925.



Brooks and Holtz's compensated transformers

Iliovici's compounded transformers

FIG. 8.

current transformers in which the errors are reduced by some compensating or compounding arrangement. Brooks and Holtz* have described such an instrument in which the transformation takes place in two stages. The first stage effects transformation in the usual way; the second stage yields a corrective current in a tertiary winding which, when combined with the secondary current of the first stage gives a sum equalling very closely what would be obtained from a perfect transformer. The combination of the currents is effected in the instrument connected to the transformer, which for this purpose must have two windings one to receive the secondary and one the tertiary current. Tests show the performance of these transformers to be almost perfect, rendering them suitable for use as precision standards.

Iliovici† has described a type of compounded current transformer embodying a different principle. He has shown that if the iron core be magnetised to the point of maximum permeability, <u>i.e.</u> to an induction of 4000-6000 lines per sq. cm. instead of the customary 500 to 1000, by an auxiliary alternating current in a winding having zero mutual inductance with respect to the primary and secondary coils, the phase-angle becomes/

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* H.B.Brooks and F.C.Holtz., "The two stage current transformer," <u>Trans.Amer. I.E. E.</u> vol 41, pp.382-391. 1922.

† M.Iliovici, "Transformateurs d'intensité compoundés," <u>Bull.Soc. Franc.Des Elecns.</u>, vol.3., pp.55-70, 1923.

becomes very small and independent of the secondary current. Such transformers have been widely adopted in France as standards and have the great advantage over the preceding type that the instruments used in conjunction therewith are of ordinary construction.

The principle of each type of compounded transformer is illustrated in Fig.8, which is self-explanatory.

6. Source of A.C.Supply.

The main source of supply for instrument transformer tests is required, especially in the calibration of current transformers, to deliver a not inconsiderable output. For this reason a suitable alternator direct coupled to a d.c. shunt motor is best, the motor preferably supplied from a battery of storage cells of adequate ampere-hour capacity. The alternator is preferably of polyphase construction with a winding tapped for 3-phase and 2-phase supply, thus giving a variety of single phase voltages as well as the polyphase voltages required by some testing methods. The alternator should give an approximately sine shaped wave-form, though this is not important, and together with its motor, must be capable of covering conveniently the whole range of frequency met with in practice, say 25 to 60 cycles per second. These requirements are, as a rule, easily met by the machines installed in most test-rooms.

Intesting voltage transformers it is usually necessary to/

to interpose a suitable step-up transformer between the test circuit and the alternator; this may conveniently be a voltage transformer similar to that under test but used in the inverse sense. Similarly, in current transformer testing the circuit is connected to the machine through a step-down transformer - an inverted current transformer often suffices - giving 4 to 8 volts at its terminals. Regulation of the current is most conveniently made by use of the field regulator of the alternator in conjunction with series resistances on the primary side of the step-down transformer. Alternatively if the secondary current thereof is not too great, regulation may be effected by an adjustable iron-cored reactance and carbon resistances in the test circuit.

For tests on site the supply must be taken from the available network, suitable transformers being interposed to bring the voltage to a value convenient for use on the test circuit. In current transformer testing on site it is often necessary to test the transformers while normally connected and to use as the testing current that flowing to the load on the network.

For all ordinary purposes in the test-room an alternator driven by a d.c. motor supplied at constant voltage runs/

runs quite sufficiently steadily. In some balance methods a vibration galvanometer is used as the detector and this instrument, as is well-known, works in resonance with the supply; if sensitivity is to be maintained it follows, therefore, that the frequency must be kept very steady. If, as is often the case, the galvanometer has more than sufficient sensitiveness at the low frequencies at which transformers are tested the influence of frequency fluctuations can be largely minimised by working with the galvanometer normally slightly shunted, thus blunting considerably its resonance peak and rendering the loss of sensitiveness with small deviations of the frequency from the tuned value of less importance. In work of very high precision, however, where the maximum of sensitivity is desired it is often necessary to hold the speed rigorously constant; means of doing this have been considered by the writer in another place* and need not be entered into here.

7. Auxiliary Sources. Phase Shifting devices.

In many test methods it is necessary not only to supply the transformer to be tested but also to apply to certain portions of the test circuit auxiliary voltages bearing a known phase relationship to that of the main supply. This is most simply done by the use of polyphase alternator/

*B. Hague, loc. cit., pp. 135-139, 1923.

alternator supplying the main circuit from one phase and the auxiliary circuit from such of the other phases as the method may require. In this way phase relationships of $\pi/2$ and of $2\pi/3$ are easily obtained between main and auxiliary supply; the phase relation is, of course, fixed and is not capable of fine adjustment. A similar result can be attained, though less easily, with a single phase alternator supplying the main circuit and some "phase-splitting" device - such as are well-known in methods of obtaining an approximate two or three-phase supply for starting of single-phase induction motors - to which the auxiliary devices may be connected at will.

In many methods a fine adjustment of the phase-displacement between the voltages applied to the main and auxiliary circuits is essential. One excellent and widely used method, applicable only in the test-room, is to use a pair of exactly similar rotating armature alternators coupled directly to one another and to a common d.c. driving motor. The field magnet of one alternator is mounted on bearings so that it can be rotated about the axis of the machine by means of a worm and wheel; by this means the voltages induced in the armatures of the two machines can be relatively adjusted, without alteration of magnitude, by an electrical phase-angle which is at once obtainable from the observed relative/

relative angular displacement between their pole systems and the number of poles in the machines. This angle can be measured by a suitable scale and vernier attached to the worm gearing, or for very precise measurement of small angular changes a mirror and scale device is easily applied. The disadvantage of the apparatus is that it requires special machines and is not portable; it should be part of the equipment, however, of any good test-room.

Another excellent method of obtaining an auxiliary voltage in any desired relation of phase to the main supply is by means of the phase-shifting transformer perfected for purposes of a.c. testing by Dr Drysdale;* this piece of apparatus is, moreover, portable and can be used not only in the test room but also on site. In principle it is identical with the well-known induction regulator and resembles an induction motor in the assembly of its parts. The stator is wound with a polyphase winding which can be connected directly to the main source of supply if this be polyphase or through a phase-splitting device if single-phase; the currents flowing in this winding set up a rotating field in the air-gap of the machine. The rotor is provided with a single phase winding and can be moved by a worm gear about its axis within the stator/

* C.V.Drysdale, "The use of a phase-shifting transformer for wattmeter and supply meter testing," Elecn., vol.62, pp.341-343, 1909.

stator bore; the phase of the voltage induced in the rotor winding by the moving field can then be adjusted to have any value relative to the main supply without the magnitude of the voltage being altered. The angle is read by a suitable scale and vernier. A sinusoidal secondary voltage is obtained by careful proportioning and suitable distribution of the stator and rotor windings.

A number of other methods,* more or less frequently employed when the preceding pieces of apparatus are not available, are also used. For example, if two slider resistances of the ordinary tubular type be connected one between lines I and II of a three phase supply while the other is joined between lines I and III the p.d. between the sliding contacts on the resistances can be easily adjusted in magnitude and phase by moving these contacts along. In a better device of this type, phase regulation with a constant voltage is obtained by the use of two coupled auto-transformers.† Another arrangement‡ consists of an iron ring closely wound with a layer of insulated wire in a way similar to a Gramme ring. The insulation is removed from the central portion of the wires on the outer circumference of/

* See G.W.Stubbings, Elec.Rev., vol.94, pp.604-606, 1924.
† C.H.Sharp and W.W.Crawford, Trans.Amer.I.E.E., vol.29, pp. 1517-1541, 1911.
‡ See W.Jaeger, 'Elektrische Messtechnik,' p.413, 1917.

of the ring; at three points 120^{0} apart on the inner perimeter of the ring tappings are taken to the three lines of a three-phase circuit. An arm, pivoted at the centre of the ring, carries two brushes which touch the bared wires on the outer circumference at diametrically opposite points. As the arm is rotated the phase angle of the voltage tapped off by the brushes is varied.

8. Indicating Instruments.

In practically all methods of transformer testing indicating instruments, ammeters, voltmeters, wattmeters, are required for the purpose of making suitable adjustments in the circuit. These should be of the ordinary laboratory portable type of dynamometer instrument, of first grade accuracy and carefully calibrated. Some relative methods make use of watt-hour meters; these are of normal single-phase construction but usually require to be specially adjusted for the present purpose. Their use is discussed in the proper places in Parts II and III.

9. Detectors.

In many methods, as already mentioned, the secondary current or voltage is opposed to a portion of the primary current or voltage, and some detecting device is required to enable the resultant to be measured. Alternatively, if as in null methods this resultant is balanced out by some auxiliary compensating/

compensating circuit, a detector will be necessary to indicate when balance has been attained. In either case such a detector is really no more than some form of a.c. galvanometer capable of detecting and measuring small alternating currents and voltages. There is a great number* of such a.c. detectors suited for use at commercial frequencies, but of these only a few have been extensively used in transformer testing. Those most favoured are (a) the separately excited dynamometer, (b) the d.c. galvanometer used with a mechanical rectifier, (c) the vibration galvanometer; these will be briefly discussed in the following subsections. In addition to these, certain special devices have been occasionally used. Of these the electrometer is the most important; owing to its very special construction and the difficulties attending its use this instrument has not been very much adopted. It is worthy of mention because at the P.T.R. in Berlin and the N.P.L. in London the electrometer forms the detector in standard high precision testing methods; it is hardly likely, however, that the instrument will be found in regular use elsewhere than in these two great national laboratories. Less important devices are the differential thermocouple with d.c. galvanometer/

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* B.Hague, <u>loc. cit.</u>, pp.145-150, 152-176, 1923.

galvanometer, and the Northrup comparator; these are dealt with in their proper context in Part II.

9a. <u>Separately excited dynamometer</u> - This instrument is the same in general construction as the ordinary dynamometer wattmeter; indeed for some of the rather less refined tests a good watt-meter serves admirably. However, to attain sufficient sensitiveness for laboratory use the pointer type of pivoted instrument is redesigned, the moving system being suspended and the deflections read by the aid of a mirror and scale or other optical device. If I be the current in the moving coil of such an instrument and I' the current in its fixed coils the reading is proportional to $II' \cos\phi$, if ϕ is the phase difference between these currents. In using the dynamometer the current I is that which is to be measured; I' is supplied from an auxiliary source, of which the phase can be regulated relative to the primary current or voltage of the transformer under test. By taking readings of the dynamometer first with the exciting current I' in phase with the primary quantity and then when at a phase displacement of θ relative thereto the components of I in these two directions can be found; θ is very conveniently 90°, giving the components of I in phase and in quadrature with the primary magnitude. The scale of the instrument can be marked off in amperes, volts or watts as desired. In most methods an error is introduced if the moving coil has any appreciable inductance; hence, every endeavour must be made to keep the moving/

moving coil circuit of low time-constant either by including a large proportion of resistance therein in comparison with the reactance, or by compensating the reactance with a shunted condenser in the way often adopted in wattmeters for precision measurements.

The dynamometer can be used in null tests by adjusting the balancing network until the instrument reads zero with I' both in phase and in quadrature with the primary quantity; this can only occur when I is zero, that is, when balance has been secured. Moreover, the instrument will be sensitive to resistance adjustments for the one position of I' and to resistance adjustments for the other; hence the deflection of the instrument indicates the amount and direction of each adjustment to be made to attain the null condition.

A great advantage of the dynamometer and of all methods in which it is used is that the direction of the deflection of the instrument serves to check the relative polarity or phase sequence of the terminals of transformer windings, which is a matter of considerable practical importance.

<u>9b. Synchronous rectifier and d.c. galvanometer</u> - This instrument is probably the most sensitive used in a.c. testing. It consists briefly of some form of rectifier, usually of a rotary commutator type,* working in synchronism with the a.c. supply, rectifying/

* F. Bedell, <u>Journal Frank Inst.</u>, Vol 176., pp.385-404, 1913.

rectifying the current to be measured, and passing it through a d.c. galvanometer; thus obtaining the advantage of the very high sensitiveness of the modern galvanometer in a.c. testing. Though simple in principle the instrument is very troublesome to use in practice, owing to the difficulty of obtaining steady contacts on the commutator, the development of thermo-electric e.m.f. thereat and so on. To overcome these troubles Sharp and Crawford* designed a cam-operated rectifying key in which rubbing contacts are abolished, thereby gaining much greater certainty of action; devices of this kind have been widely used by American experimenters.

The effect of phase shifting of the auxiliary current in a separately excited dynamometer is obtained in the present instrument by setting the brushes or operating cam so that rectification occurs at different points in the cycle. When used in null measurements the instrument again enables discrimination to be made between the necessary resistance and reactance adjustments, permitting one to be adjusted independently of the other.

9c. Vibration galvanometer. - The well-known vibration galvanometer is widely used in null methods of transformer testing, for which work it is particularly well suited since at commercial frequencies the instrument has a very high sensitiveness. Unlike the preceding instruments the vibration galvanometer does not give by its deflection any indication/

*C.H. Sharp & W.W. Crawford, Trans. Amer. I.E.E. vol.29, pp.1517-1541, 1911.

indication of the nature or direction of the adjustments to be made to approach balance. This disadvantage is of much less importance than would be supposed, since a very little experience and systematic use of the adjustments enables balance to be very quickly secured. Indeed it is probably the most rapid instrument of all to use in null tests. The construction, use, advantages, and theory of the galvanometer have been fully treated by the author in the place cited and need not be further dealt with here.

10. Secondary burdens.

The burden of an instrument transformer is the external load connected to the secondary when the transformer is in operation; the amount and nature of the burden have a great influence upon the ratio and phase of the transformer. Consequently, when transformers are tested, care must be taken to make the tests with a burden equivalent to that with which the transformer is to be used. Often this is not easy to arrange ~~secure~~ since all methods of testing involve the inclusion of some part of the testing apparatus in the secondary circuit. In some methods the burden thus imposed is slight. and may be neglected in comparison with that of the actual instrument which is the transformers' normal load. In other methods this is by no means the case and the transformer must then be loaded with an artificial burden. This takes the form of resistances/

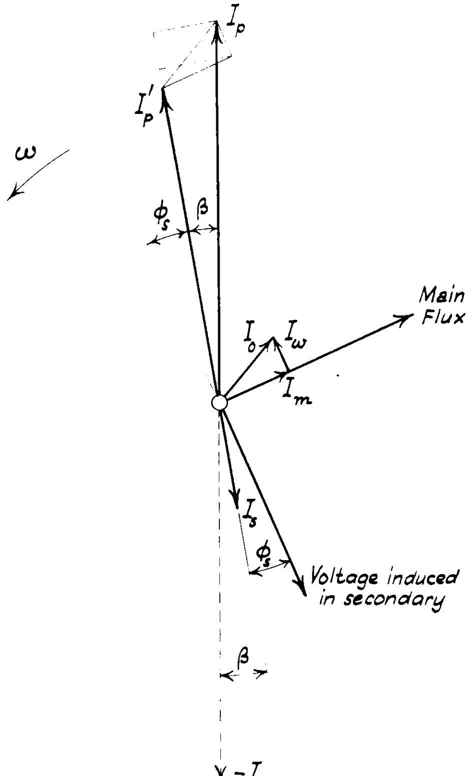
resistances and inductances, capable of carrying 5 amperes, combined to give in conjunction with the testing apparatus an impedance equal to that of the load with which the transformer will work. As a guide to the proper choice of burdens the following table gives the constants for a few typical voltmeters, ammeters and wattmeter current coils; each test-room will acquire for itself data concerning the instruments normally encountered and prepare therefrom the necessary artificial loads.

Instrument.	Resistance. Ohms.	Inductance Henrys.	At 50 cycles per sec. Impedance ohms.	Power Factor.	Volt-Amperes.
...er & Thomson ...t iron Voltmeter.	1996	0.45	2000	0.998	6.05 at 100 volts
...ton Soft Iron Voltmeter.	685	0.07	689	0.995	17.55 at 110 volts
...ton Dynamo-...er Voltmeter.	1145	12.6×10^{-3}	1146	0.999	10.55 at 110 volts
...d H. Dynamo-...er Ammeter	0.55	0.04×10^{-3}	0.55	1	13.75 at 5 amp.
...d B. Hot wire Ammeter.	0.05	Negligible.	0.05	1	1.25 at 5 amp.
...d H. Wattmeter current coil.	0.165	0.33×10^{-3}	0.195	0.845	4.88 at 5 amp
...nd B. Wattmeter ...rent coil.	0.125	0.50×10^{-3}	0.201	0.621	5.02 at 5 amp

PART II.

TESTING OF CURRENT.

TRANSFORMERS.



I_p
I_p'
ω
ϕ_s
β
Main Flux
I_0
I_w
I_m
I_s
ϕ_s
Voltage induced in secondary
β
$-I_p$

FIG. 9.

CHAPTER I.

INTRODUCTORY.

------

1. Theory of the current transformer.

It is now necessary to look somewhat more closely into the theory of the current transformer than was done in Part I, so far as is required to enable the various characteristics and their measurements to be understood. Fig. 9 shows the vector diagram for a current transformer, drawn in its simplest form* to show the current relationships. The secondary winding is supposed to be closed through a given external burden consisting of some instrument or appliance to be operated by the transformer; this burden together with the impedance of the winding itself constitutes the total secondary load of phase-angle ϕ_s . If now a certain primary current be applied there will be a drop of voltage over the primary winding, necessitating the production of a flux in the core sufficient to set up the primary back e.m.f. This flux, being linked with the turns of the secondary winding/

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* See P.G.Agnew, "A study of the current transformer with particular reference to iron loss," Bull.Bur.Stds., vol.7, pp.423-474, 1911. For an exhaustive study see E.Rosa and M.G.Lloyd, "The determination of the ratio of transformation and of the phase relations in transformers," Bull.Bur.Stds., vol.6, pp.1-30, 1910, where a variety of secondary matters are discussed. Consult also E.L.Wilder, "Operation of the series transformer," Elec.J., vol.1,pp.451-455, 1904; K.L. Curtis, "The current transformer," Trans.Amer.I.E.E., vol.25, pp.715-734, 1907; C.V.Drysdale, "The use of shunts and transformers with alternate current instruments," Phil.Mag.,5th ser.,vol.16,pp.136-153,1908; E.S.Harrar, "The series transformer," Elec.World, vol.51,pp.1044-1046, 1908.

winding, will induce therein a voltage which causes a current I_s to circulate in the secondary cirquit ; the magnitude I_s and phase ϕ_s of the secondary current relative to the induced secondary voltage are determined by the reactance and resistance of the whole secondary circuit, winding and burden inclusive. If magnetic leakage be neglected* the primary current must contain a component opposite in phase to I_s and of a magnitude equal to $K_T I_s$, where K_T is the ratio of the secondary to the primary turns; i.e.,

$$I_p' = K_T I_s = \frac{T_s}{T_p} I_s$$

or $$I_p' T_p = I_s T_s$$

that is, the secondary load ampere-turns are balanced by the primary load ampere-turns.

A certain number of primary ampere-turns are required to produce the flux in the core, these being represented by a wattless magnetising component I_m of the primary current drawn in phase with the flux. In addition there will be a current component perpendicular to the flux, I_w , accounting for the loss of energy by hysteresis and eddy currents in the iron core. The total exciting current I_o required for the magnetisation of the core is the vector sum of I_m and I_w . The primary current I_p is compounded of the load component I_p' and the exciting component I_o , as shown. The ratio of/

* This is examined by M.Rosenbaum, "The current transformer," Elecn., vol.74, pp.626-630, 1915, and is shown to lead to modifications in the theory that are of secondary importance.

of the transformer is then $K_c = I_p/I_s$ and the phase-angle β.

It must be remembered that unlike the voltage transformer, or power transformers, the current transformer operates at variable primary voltage. Every change in the primary current causes a different applied p.d. to be impressed on the primary winding, necessitating a corresponding change in the flux and with this in the exciting current. It follows, therefore, that both K_c and β are functions of the amount and phase of the secondary current and of the exciting current I_o required to produce the flux; in other words, K_c and β depend on I_s, ϕ_s, I_m and I_w. Quantitative relations will be considered in the next section, but it follows at once with the magnitudes related as in Fig.9 that the true ratio $K_c = I_p/I_s$ exceeds the turns ratio $K_T = T_s/T_p$.

2. <u>Analytical Theory</u>.

The vector diagram of Fig.9 readily lends itself to analytical treatment. Resolving the current vectors on and normal to I_p' gives

$$I_p \cos\beta = K_T I_s + I_m \sin\phi_s + I_w \cos\phi_s$$

$$I_p \sin\beta = I_m \cos\phi_s - I_w \sin\phi_s.$$

Squaring and adding, remembering that I_m and I_w are usually small enough for their products and squares to be neglected, gives

$$I_p^2 \doteq K_T^2 I_s^2 + 2K_T I_s (I_m \sin\phi_s + I_w \cos\phi_s)$$

whence /

whence

$$K_c \doteq \frac{I_P}{I_S} \doteq K_T\left[1+\frac{2}{K_T I_S}\left(I_m \sin\phi_S + I_W \cos\phi_S\right)\right]^{\frac{1}{2}}$$

or

$$K_c \doteq K_T + \frac{I_m \sin\phi_S + I_W \cos\phi_S}{I_S}.$$

This shows that in general the true ratio is greater than the turns ratio by an amount depending on the components of exciting current and the magnitude and phase of the secondary load.

For the phase-angle β, dividing the original equations,

$$\tan\beta = \frac{I_m \cos\phi_S - I_W \sin\phi_S}{K_T I_S + I_m \sin\phi_S + I_W \cos\phi_S} \doteq \frac{I_m \cos\phi_S - I_W \sin\phi_S}{K_T I_S}$$

Relationships similar to these have been given by a number of authors* and form the basis of the indirect methods of testing transformer characteristics; they will be further considered in Chapter II.

At a high power-factor on the secondary the ratio is principally affected by I_W and the phase-amgle by I_m. Thus for example at unity power-factor, $\phi_S = 0$ and

$$K_c \doteq K_T + \frac{I_W}{I_S},$$

and also $\tan\beta \doteq \dfrac{I_m}{K_T I_S}.$

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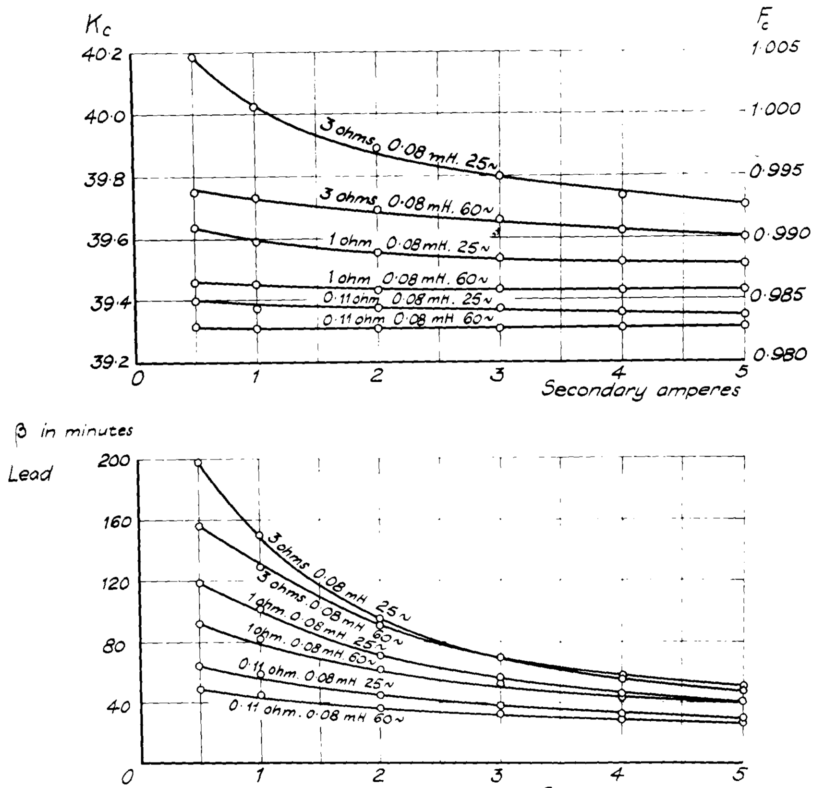
* K.L.Curtis, loc.cit.,1907; C.V.Drysdale,loc.cit.,1908; W. Gehkin, "Sur transformateurs d'intensité,"Lum.Elect.,vol.8, 2nd ser.,pp.67-71,1909; A.P.Young, "The theory and design of current transformers," Journal I.E.E.,vol.45,pp.670-678,1910; L.T.Robinson,loc.cit.,1910; A Barbagelata,"Prova indiretta dei trasformatori di misura per forti intensità di corrente," Atti dell' Assoc.Elett.Ital.,vol.14,pp.639-654,1910; P.G.Agnew, loc.cit.,1911; A.G.L.McNaughton, "The current transformer," Journal I.E.E., vol.53, pp.269-271, 1915.

In many cases the secondary load may have a very low power-factor, e.g., when the burden is a trip coil or even an induction instrument. Then making the power-factor zero

$$K_c \doteqdot K_T + \frac{I_m}{I_s},$$
$$\tan\beta \doteqdot -\frac{I_w}{K_T I_s},$$

are the limiting values, showing that the ratio is now determined by I_m and the phase angle by I_w. Moreover, it should be observed that at low power-factors in the secondary circuit β may reverse in sign, i.e., if originally I_s were leading on I_p reversed it now lags.

It is relevant to speak here of the practical problem of reducing the imperfections of a transformer to a minimum. It is obvious that K_c will only be identical with K_T for all secondary burdens if I_m and I_w are zero. Similarly β will be zero only if I_m and I_w both vanish. This condition can never be attained with any practical magnetic material; hence it is necessary to use the best possible iron for the magnetic circuit. By making a short, jointless, magnetic circuit of highly permeable iron alloy, working at a flux density not exceeding 1500 lines per sq.cm., I_m can be made very small; the reduction of I_w is effected by choosing a material with low hysteresis loss and a high specific resistance, so that when used in thin laminae the eddy current losses in the iron become very small. Drysdale has recently pointed out that a considerable improvement in characteristics could/



K_c
F_c
3 ohms 0·08 mH. 25~
3 ohms 0·08 mH. 60~
1 ohm 0·08 mH. 25~
1 ohm 0·08 mH. 60~
0·11 ohm 0·08 mH. 25~
0·11 ohm 0·08 mH 60~
Secondary amperes
β in minutes
Lead
3 ohms 0·08 mH 25~
3 ohms 0·08 mH 60~
1 ohm. 0·08 mH 25~
1 ohm. 0·08 mH. 60~
0·11 ohm. 0·08 mH 25~
0·11 ohm. 0·08 mH 60~
Secondary amperes

FIG. 10.

could be obtained by substituting for the materials of the Stalloy type one of the new high permeability metals such as Permalloy.

3. Typical values of ratio and phase-angle.

It will be appreciated from the preceding discussion that very great care is required in the design and construction of current transformers if ratio and phase-angle errors are to be made small; a great diversity in behaviour is found among transformers constructed by various makers. The curves in Fig.10, plotted from the classic results of Agnew and Fitch,* show how the characteristics of a current transformer depend on secondary current, burden, and frequency in a normal case. The curves refer to a transformer of 200/5 amp. ratio, nominal ratio 40/1, rated at 40 volt-amperes; the transformer has a secondary resistance of 0.463 ohm and is insulated for 15,000 volts. It will be seen that in a general way the ratio and phase-angle curves are of similar forms, falling with increasing secondary current and concave upwards. The actual numerical values are perhaps somewhat higher than occur in modern precision transformers but represent quite closely the behaviour of a good transformer. The phase-angle β is usually positive, i.e., I_s leads on $-I_p$, but at heavy inductive load β may become negative or lag. It is to be noted in passing that in deference to custom β is stated in minutes; this is a quite indefensible practice/

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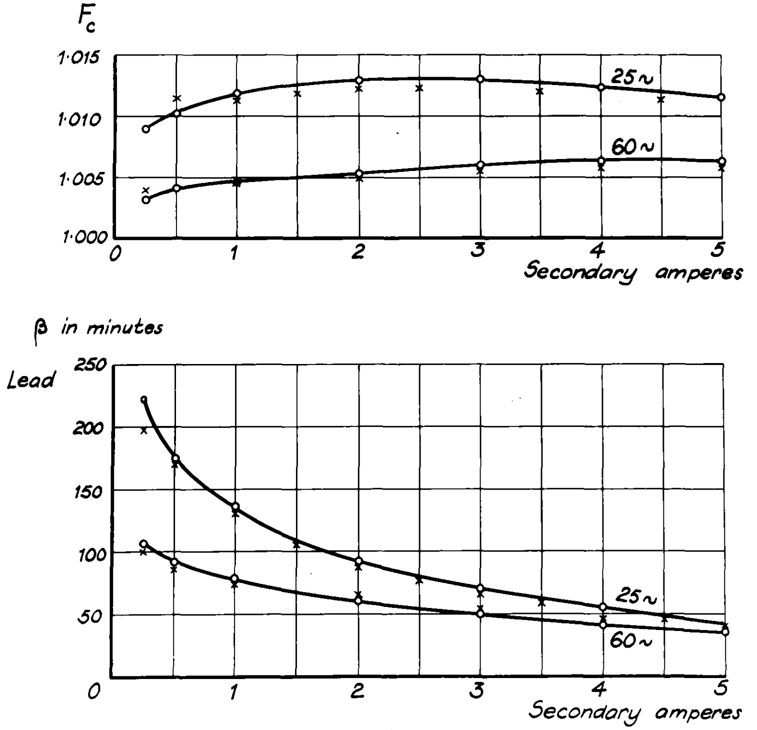
* P.G.Agnew and T.T.Fitch, Bull.Bur.Stds., vol.6, pp.281-299, 1910.

practice* since in all practical instances where β is required in calculation it must be converted into radians. This is readily done by dividing the angle in minutes by 3438. Since β is usually small it would be a distinct advantage to keep it in radians, for then the sine and tangent which are generally required could without appreciable error be taken equal to the angle itself, thus rendering unnecessary the use of trigonometrical tables. The value of β , even in transformers intended for use with portable instruments, may at low loads be in excess of 3°, but with careful design should be much less.

Jolley† has recently compared the behaviour of two distinct types of current transformer, one of American (A) and the other of British (B) manufacture. Both were of 50/5 ratio, A for 25 volt-amp. and B for 15 volt-amp. The ratio between 1/5 load and full load in both transformers changed by little more than 1%. The American transformer had a jointless core of very good iron worked at 730 lines per sq.cm.; the British transformer had interleaved joints, inferior iron, and a flux density of 3940 lines per sq.cm. The phase-angle of B was negative, decreasing with load, while that of A was positive and increasing. The negative angle of B never exceeds 15 min. and is always numerically less/

* A.Campbell, "Angular unit for small phase differences," Journal Sci.Insts., vol.3,pp.93-94, 1925.

† A.C.Jolley, "Some tests on modern current transformers," Journal Sci.Insts., vol.3,pp.43-50, 1925.



o Observed points
x Calculated points

FIG. 11.

less than that of A; this small negative angle is attributed to the higher reactance and core loss of B.

4. Abnormal ratio curves.

While the curves discussed in Section 3 and shown in Fig.10 represent the general trend of results obtained in normal transformers, certain abnormal results are sometimes encountered the investigation of which sheds much light on the nature of ratio and phase errors in general. Such results were first noticed by Edgcumbe* in 1910 but no explanation was offered; this was supplied by Agnew† in the following year.

Fig.11 is taken from Agnew's memoir and illustrates the effect very clearly. The phase-angle follows the normal course, decreasing with increasing secondary current. The ratio, on the other hand, increases to a maximum before finally decreasing as full load is approached. Such a ratio curve is quite abnormal. The transformer on which these curves were taken had a turns ratio T_s/T_p = 196/25, nominal ratio 40/5 amperes, secondary resistance 0.51 ohm, secondary burden 0.17 ohm resistance and 0.08 millihenry inductance, maximum flux 290 lines at 60 cycles and 700 lines at 25 cycles.

Agnew shows that the form of the ratio curve depends entirely upon the way in which the iron losses vary with the/

* K.Edgcumbe, "Some notes on the use of instrument transformers," Elec.Rev., vol.67, pp.163-165, 1910.

† P.G.Agnew, loc.cit., 1911.

the maximum flux density B in the core. Thus if the loss be taken proportional to B^c, the curve will be of normal form sloping down and concave upwards if $c < 2$. If $c = 2$ the curve will be horizontal, as is approached in some high-class transformers with low impedance load on the secondary. If, however, c lies between 2 and 3 the curve will slope up with downward concavity, as in the abnormal form illustrated. Agnew was able to justify these deductions by an extensive series of experiments undertaken to investigate the variation of the loss index c with B at low flux densities in materials such as are used in current transformer core construction. He shows that c is far from constant under such conditions, but demonstrates that the slope of the ratio curve can be predicted with accuracy from the slope of the curve obtained by plotting the core loss against the flux on logarithmic paper, i.e., on the logarithmic slope or "ratio of variation" of the core loss. Fig.11 will be again referred to in Chapter II.

5. Variation of characteristics with working conditions.

The ratio and phase-angle of a current transformer depend very considerably upon a number of factors introduced by the working conditions; the variations are best displayed by plotting the ratio or ratio-factor and the phase-angle to a base of secondary current for various conditions, vide Fig.10. The variables affecting the performance of the transformer/

transformer are (a) frequency, (b) secondary burden, (c) magnetic history, and (d) wave-form. These will now be briefly examined.

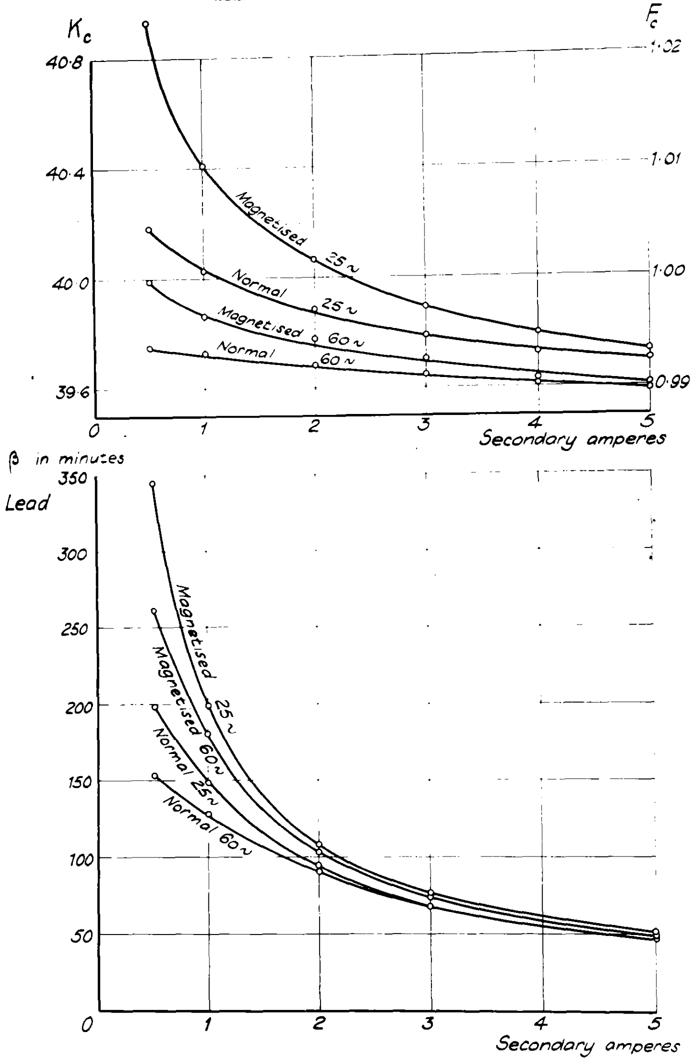
5a. Frequency. - For a given current through a given secondary burden a definite voltage, proportional to the product of the frequency and the flux in the core, is required. Hence if the frequency be lowered the flux will be correspondingly increased. A larger exciting current is therefore taken by the primary winding, the general effect being usually to increase the ratio K_c and the phase-angle β in a transformer of normal characteristics. Moreover, the lowering of the frequency increases not only the actual values of K_c and β but also the rate with which these quantities vary with I_s.

The effect of frequency is considerable, especially on the value of the angle β, and hence the frequency should always be clearly stated in any tests on the characteristics of a current transformer.

5b. Secondary burden. - An increase in the impedance of the secondary burden calls for an increase in the induced secondary voltage if a given secondary current is to be maintained at a given frequency. This increased voltage requires a proportionate increase in the flux. Hence in general an increase of secondary burden has the same influence as a decrease of frequency, increasing the flux for a given secondary current and in consequence of the larger exciting current/

current, increasing both the ratio K_c and the phase-angle β. The precise effect of a change in secondary burden depends not only upon the total change of impedance, or of volt-amperes, but on the resistance and reactance alterations that have been made. It is not sufficient in transformer testing merely to follow the common practice of stating the impedance of the secondary burden or the number of volt-amperes absorbed by it; the resistance and reactance of which it is composed must be clearly specified if the figures given for ratio and phase-angle are to have a definite meaning. In testing, therefore, it is essential to see that tests are made with a real or an artificial burden in the secondary circuit which imitates precisely that with which the transformer will work in practice; the statement of the constituents of the burden as well as its total amount is absolutely essential for definiteness.

It is perhaps well to point out here that the current transformer used in conjunction with an instrument must be looked upon as part of that instrument and partaking of the precision demanded therefrom. The practice, still not uncommon, of using a current transformer to supply not only a measuring instrument but also such devices as protective relays, trip coils, etc. cannot be too strongly deprecated, since it is impossible to expect that a transformer can have sufficiently good characteristics to enable it to act with precision as a measuring device and simultaneously as a piece/



Influence of d. c. magnetisation on the characteristics given in Fig. 10.

FIG. 12.

piece of protective apparatus. Such mixed burdens cannot be too strongly discouraged; each instrument should have its own transformer of adequate precision, while each detail of protective gear should be operated from a low accuracy transformer of sufficient secondary output.

5c. Magnetic history. - It is well known that the secondary circuit of a current transformer should never be opened when the primary winding is excited. If it were opened the secondary back ampere-turns would be removed and the core would be magnetised to a high flux density; this would result in greatly increased iron losses with consequent influence on the ratio and phase-angle. A precisely similar effect is obtained if a direct current of full-load value had been passed round either winding, as is sometimes done in testing the windings for relative polarity. The influence of such a d.c. magnetisation is shown in Fig.12, plotted from the results of Agnew and Fitch, indicating the great increase in ratio and phase-angle produced by the excessive exciting current taken by the initially magnetised transformer. The effect of open-circuiting the secondary is similar, the amount of increase in K_c and β depending upon the point in the cycle at which the primary circuit is opened, i.e., whether the core were left more or less magnetised.

The effects of magnetisation due to either cause can be readily overcome by opening the secondary, passing a current slightly in excess of full load current through the/

the primary, and then gradually reducing the current to zero. Demagnetisation may often be more conveniently effected by opening the primary and passing an alternating current through the secondary, reducing it gradually to zero as before. All transformers should be thoroughly demagnetised before being tested for ratio and phase error, preferably by the second method. Engelhardt* shows that a current of 0.2 ampere is usually sufficient for the purpose. He found in certain transformers that remanent magnetism increases the ratio at full load by 0.1 to 0.3% and the angle by as much as 14 min. At 1/10 full load a ratio error of 1.8% and an angle error of 85 min. was observed.

5d. Wave-form. - For all practical purposes the influence of the wave-form of the primary current upon the ratio and phase-angle of a current transformer is negligible. The question was originally investigated in 1896 by Roessler† for power transformers, and in 1908 by Lloyd‡ for the more special case of instrument transformers. This experimenter shows that the influence of wave-form depends largely upon whether/

* V.Engelhardt, "Ueber den Einfluss der remanenten magnetisierung auf die Angaben von Stromwandlern und über deren Beseitigung." Elekt.Zeits.,vol.41,pp.647-650, 1920.

† G.Roessler, "The behaviour of transformers under the influence of alternating currents of different wave-forms." Elecn., vol.36,pp.124-126,150-153,184-185,219-222, 1896.

‡ M.G.Lloyd, "The effect of wave form upon the voltage ratio of transformers," Elec.World,vol.52,pp.845-846,1908; "Effect of wave-form upon iron losses in transformers," Bull.Bur.Stds., vol.4,pp.477-510, 1908. See also P.G.Agnew, loc.cit.ante, 1911.

whether the primary resistance drop preponderates over the leakage reactance drop or the reverse. With predominating ohmic drop a peaked wave reduces the ratio; with predominant reactance drop a peaked wave causes the ratio to increase. In current transformers the effects are negligible unless the harmonic exceed half the fundamental; the magnitude of the effect is of the same order as that due to small changes of frequency, a peaked wave reducing both ratio and angle.

6. Wave distortion.

The question of how far the wave of secondary current of a current transformer is a reduced facsimile of the wave of primary current is one that has often been asked. Robinson* by taking oscillograms of the primary and secondary currents showed that any difference, even with a very distorted primary wave, must be exceedingly small. The later experiments of Agnew, making use of refined methods of wave analysis, have shown the effect to be quite negligible in all practical cases. With a 20% harmonic in the primary current the distortion in the secondary wave amounted only to 1 part in 2500, and would in practice be much less with the purer wave-forms usually encountered. Hence the primary and secondary currents can be considered, even in the most refined work, to be of identical wave-form.

7. Introduction to methods of testing.

Having now discussed the characteristic properties of current transformers the following chapters of this Part will be/

* L.T.Robinson, loc.cit.ante.,1910; P.G.Agnew, loc.cit., 1911.

be devoted to a résumé of the principal methods of measuring the ratio and phase-angle that have from time to time been suggested. Some of these methods are of historical interest only, others are of the greatest practical value but they are all classified on a purely technical basis in the present Part. The task of separating out the methods that would nowadays be used from the great mass of those suggested is the object of Part IV.

In Chapter II of Part I a broad general classification of methods of testing transformers was given. It is now necessary slightly to amplify that classification in its particular application to tests on current transformers. The Indirect Method is first treated in Chapter II now following. Chapters III to VI inclusive deal with Direct methods both Absolute and Relative. The methods are classified throughout according to the type of apparatus that must be essentially used in them; this seems to be simpler, to make fewer classes and sub-classes, and to attain a greater measure of correlation than by any other process.

The Absolute Deflectional methods of Chapter III utilise, in the main, two principles; (i) in which the primary and secondary currents are separately measured and compared by the readings of certain instruments; (ii) in which the volt-drops over four-terminal resistances inserted in primary and secondary circuits are adjusted to approximate balance or equality, their difference being measured by the readings of/

of suitable instruments. The great majority of these methods are based on this opposition or balance principle; in some cases the method becomes "semi-null," since one instrument is adjusted to give a zero reading before the others are read.

The Absolute Null methods of Chapter IV are essentially opposition or balance methods in which the vector resultant of the primary and secondary volt-drops is exactly annulled by an auxiliary voltage of adjustable magnitude and phase, balance being indicated by a suitable detector. These methods are, as it were, applications of the principle of an a.c. potentiometer of a limited voltage and angular range.

In Chapters V and VI Relative methods, respectively Absolute and Null, are discussed. The former are essentially opposition methods of comparing the secondary currents of two transformers, the difference between which is measured deflectionally. The latter are likewise opposition methods in which the difference of the two secondary currents or volt-drops is annulled by an adjustable auxiliary circuit.

With these preliminary remarks it is now possible to proceed to a detailed analysis of each class of methods.

CHAPTER II.

THE INDIRECT METHOD.

----

1. Preliminary and Historical.

In Section 2 of the preceding Chapter it has been shown that the ratio and phase-angle of a current transformer are very approximately given by the equations

$$K_c \doteq K_T + \frac{I_m \sin\phi_s + I_W \cos\phi_s}{I_s},$$

$$\tan\beta \doteq \frac{I_m \cos\phi_s - I_W \sin\phi_s}{K_T I_s}$$

where K_T is the ratio secondary turns/primary turns, I_m and I_W are the magnetising and loss components of the primary current, I_s is the secondary current, and ϕ_s is the phase displacement of the entire secondary circuit. If these various quantities can be determined, then K_c and β are found from the above expressions.

In the early days this method was the only one used in current transformer testing, the process following very closely the methods of testing power transformers for regulation. Thus Wilder* in 1904 measured the exciting current of a transformer with the secondary open and from these values and the open-circuit secondary Wolts calculated approximately the variation of ratio with various secondary resistance loads.

A/

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* E.L.Wilder, Elec.J., vol.1, pp.451-455, 1904.

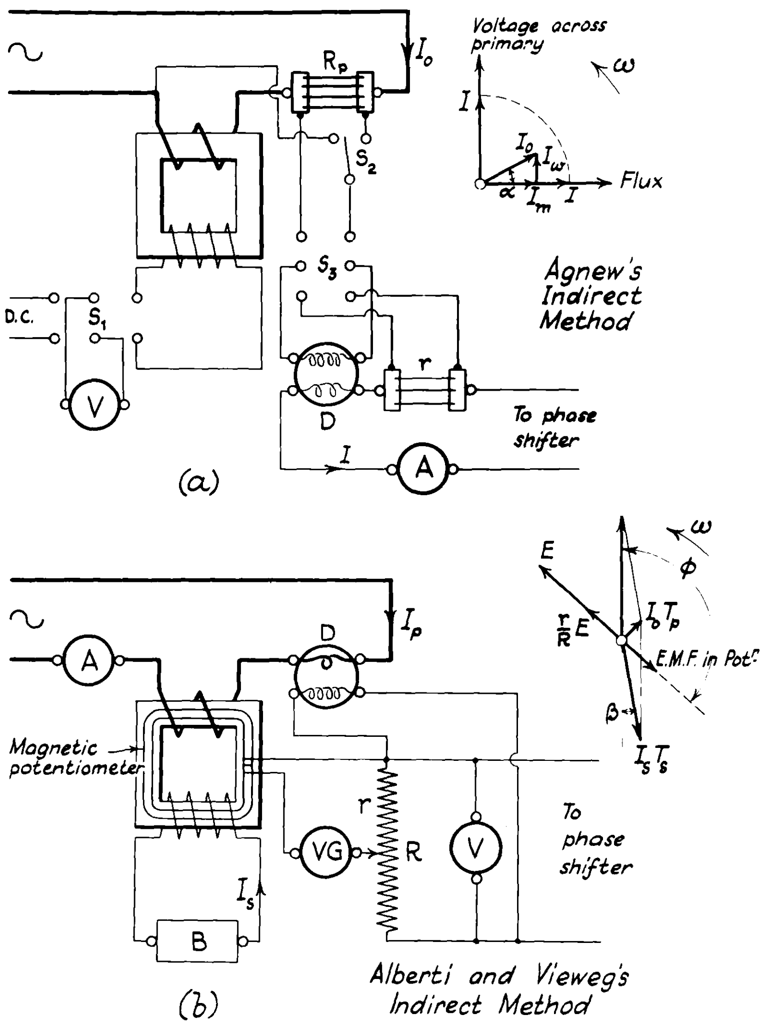
A similar but more precise use of the same method was made by McNaughton* in 1915. This experimenter measured the exciting current I_0 and its phase, hence determining I_m and I_w, by opening the secondary circuit and putting an ammeter, a voltmeter and a wattmeter in the primary, i.e., by the ordinary o.c. test. By closing the secondary through an ammeter and supplying the primary at a low voltage, measurements of primary volts, amperes and watts enable the effective impedance, resistance, and reactance of the transformer to be found at various secondary currents. It is then supposed that the secondary winding is responsible for half the effective reactance, a supposition by no means correct.

Curtis† in 1907 endeavoured to make an application of the method to a ring type transformer by first measuring the reactance and resistance of the secondary burden, and approximately calculating the leakage reactance of the secondary. By this means ϕ_s could be estimated with some exactness. The values of I_m and I_w were deduced, however, from a ballistic test of the ring, but these are far from being the correct values when the core is magnetised by an alternating current.

These particular forms of the indirect test were soon supplanted by simpler and more exact direct methods of measuring ratio and phase-angle. Interest in the indirect method was, however, again revived a few years later when it/

* A.G.L.McNaughton, Journal I.E.E., vol.53, pp.269-271, 1915.

† K.L.Curtis, Trans.Amer.I.E.E., vol.25, pp.715-734, 1907.



Voltage across primary
ω
I
I_o
I_ω
Flux
α
I_m
I
R_p
I_o
S_2
S_3
Agnew's Indirect Method
D.C.
S_1
V
r
D
To phase shifter
I
A
(a)
ω
E
φ
$\frac{r}{R}E$
I_oT_p
E.M.F. in Pot^r
β
I_sT_s
I_p
D
A
Magnetic potentiometer
r
VG
R
V
To phase shifter
I_s
B
(b)
Alberti and Vieweg's Indirect Method

FIG. 13.

it became necessary to test transformers for very large primary currents. The direct method suffers then from the technical difficulty of constructing non-reactive low resistances suitable for currents of 2000 amperes or more and the use of the indirect process enables this trouble to be surmounted. The procedure adopted[*] is to measure I_m and I_W by a dynamometer method, a typical arrangement being described in the next Section. The exact determination of ϕ_s is, however, subject to some uncertainty owing to the difficulty of determining the secondary leakage reactance; fortunately this is often small and is swamped by the reactance of the secondary burden. As a general rule excellent agreement is found between the computed and the actual values of the ratio and phase-angle, as an example in Section 2 will show.

2. Typical Method.

The following is a typical procedure for use of the indirect method, introduced by Sharp and Crawford but later improved by Agnew; the connections for the measurement of I_m and I_W are shown in Fig.13a. The secondary voltage from which the flux can be calculated, was set by means of a sensitive reflecting dynamometer voltmeter V which could be calibrated on d.c. by throwing S_1 to the left. The primary exciting current is passed through a suitable four-terminal/

* See L.T.Robinson, Trans.Amer.I.E.E.,vol.28,pp.1005-1039, 1910; A.Barbagelata, Atti dell' Assoc.Elett.Ital.,vol.14, pp.63-654,1910; C.H.Sharp and W.W.Crawford, Trans.Amer.I.E.E., vol.29,pp.1517-1541,1911; P.G.Agnew,Bull.Bur.Stds.,vol.7, pp.423-474,1911; A.Barbagelata, L'Elettro., vol.8, pp. 165-175, 1921.

terminal resistance R_p. The procedure is then as follows:- With S_3 up and S_2 to the left the phase shifter is adjusted until D reads zero, showing that the voltage across the primary winding and the current I in the fixed coil of D are in quadrature. By throwing S_2 to the right a reading of D is obtained proportional to $I I_o \cos\alpha$ i.e., to I_m. The phase of I is then advanced by 90^o and the new reading of D, proportional to $I I_o \sin\alpha$ or I_w, is taken. By this means curves of I_m and I_w as functions of the secondary voltage are obtained. Knowing the impedance of the secondary burden, the resistance of the secondary winding, and estimating the reactance of the latter if necessary, it is possible to find ϕ_s and also the induced secondary voltage required to circulate I_s secondary amperes; from this the two components of exciting current may be taken from the test curves. If the turns ratio K_T be known then K_c and β are at once calculable from the preceding equations.

To show the agreement possible between the computed and the actual characteristics the reader should refer to Fig.11 in which the crosses represent the values found by the above indirect method while the circles show the values measured by an absolute method. This is the case of a transformer with an abnormal ratio curve and hence the agreement is the more gratifying; Agnew and others give results for normal transformers in which the correspondence is no less good. As mentioned earlier the principal defect of/

of the method is the difficulty of exactly estimating the secondary leakage reactance, but this makes but a small error if the external burden be at all large, as is generally the case in practice.

3. <u>Use of the magnetic potentiometer</u>.

Alberti and Vieweg* have used quite a different method for determining the components of exciting current, namely, by means of the magnetic potentiometer. This simple piece of apparatus, introduced by Chattock† in 1888, consists of a uniform coil of wire wound on a flexible core of cord, rubber, or pressboard, and connected to a ballistic galvanometer. If the two ends of the coil be at points in a magnetic field between which there is a difference of magnetic potential, a throw of the galvanometer will result when the field is removed. This deflection will be proportional merely to the difference of magnetic potential and independent of the contour of the core upon which the coil is wound. Again if the coil be linked through a circuit which carries a current and its ends are brought into contact, when the current be stopped a throw of the galvanometer will ensue proportional to the m.m.f. round the contour of the core of the potentiometer, *i.e.*, to 4π times the ampere-turns of the circuit. If the current/

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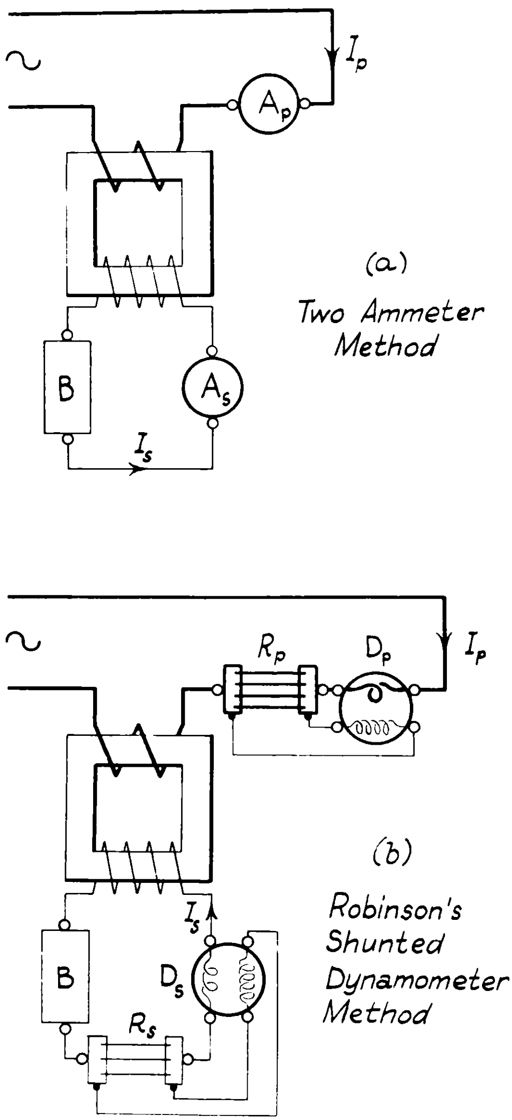
* E.Alberti and V.Vieweg, "Untersuchung an Stromwandlern. Der Magnetisierungsstrom", *Arch.f.Elekt.*, vol.2, pp.208-216, 1914.

† A.P.Chattock, "On a magnetic potentiometer," *Proc.Phys.Soc.*, vol.9, pp.23-26, 1888. See also, W.Rogowski and W.Steinhaus, *Arch.f.Elekt.*, vol.1, pp.141-150, 1913; W.Rogowski, *idem*, pp. 511-527, 1913; F.Goltze, *Arch.f.Elekt.*, vol.2, pp.303-313, 1914.

current be alternating and the potentiometer be joined to a vibration galvanometer, a deflection will be produced proportional to the ampere-turns with which the potentiometer is linked.

This is the principle upon which Alberti and Vieweg's method is based. The flexible magnetic potentiometer is linked through the primary and secondary windings of the transformer and its ends brought together, see Fig.13b. The instrument therefore measures the resultant m.m.f. of the primary and secondary ampere-turns when the transformer is supplied with current and the secondary is closed through a given burden, i.e., the exciting ampere-turns $I_0 T_p$. The voltage induced in the potentiometer is opposed against the drop of voltage in a resistance R supplied with an auxiliary current from a phase shifting device; the tapping on the resistance and the phase shifter are regulated until balance is indicated by a vibration galvanometer, the amount of the voltage being then known by the tapping fraction r/R and its phase ϕ relative to the primary current of the transformer by the readings of an ammeter and dynamometer wattmeter in the primary circuit, the volt coil of the wattmeter being excited at known voltage E from the phase-shifter. The resultant voltage in the magnetic potentiometer is thus known in magnitude and phase and can be drawn in a vector diagram in proper relation to the primary ampere-turns, which are known. The exciting ampere turns are at right angles/

angles to the resultant voltage, thus giving the direction of I_0. Moreover the secondary ampere turns are known in magnitude, so that I_0 is readily found. The results are found to be in good agreement with the values obtained from a direct absolute method.



I_p
A_p
(a)
Two Ammeter Method
B
A_s
I_s
R_p
D_p
I_p
(b)
Robinson's Shunted Dynamometer Method
I_s
B
D_s
R_s

FIG. 14.

CHAPTER III.

ABSOLUTE METHODS. DEFLECTIONAL.

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1. Two ammeter methods.

The ratio of a current transformer is most easily found by the use of two ammeters,* A_p and A_s, connected respectively in the primary and the secondary circuits as shown in Fig.14a. The ratio is then given by the ratio of the readings of primary and secondary ammeters. The method suffers from several disadvantages which render it of little use in practice, except for quite rough tests. The accuracy clearly depends upon the accuracy of calibration of the two instruments and the precision with which simultaneous readings can be taken upon them; at the full rated current the ratio may be determined with care to ½%. The scales of most a.c. ammeters are usually non-uniform, the divisions being crowded at the lower readings; hence at low currents the accuracy falls off very considerably. The accuracy at low loads cannot be improved by the substitution of a lower reading ammeter in the secondary, since the burden imposed by the windings of an ammeter of less than 5 amperes range is too great and would make the ratio of transformation quite different from the true value. The test burden B, therefore, should be chosen so that together with the secondary ammeter the/

* See R.S.J.Spilsbury, Beama, vol.6 pp.505-513, 1920, F.A. Kartak, Elec.World, vol.75,pp.1368-1370, 1920; F.B. Silsbee, Trans.Amer.I.E.E., vol.43, pp.282-294, 1924.

the total secondary burden is equal to that with which the transformer will be loaded in service. The method is limited in range by the fact that self-contained a.c. ammeters for use in the primary circuit with currents exceeding 500 amperes are not readily procurable.

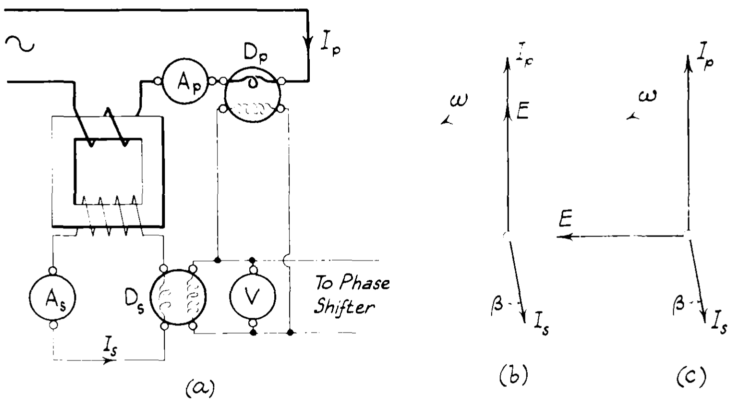
The same principle applies to the use of shunted dynamometers, as suggested by Robinson* and shown in Fig.14b. The current coil of D_p is put in the primary circuit and that of D_s in the secondary; the voltage coils are connected in parallel with suitable four-terminal resistances, R_p, R_s in the respective circuits. By the use of reflecting dynamometers it is possible to attain fairly high accuracy; high primary currents can be dealt with by proper choice of R_p and parallel grouping of the current coils of D_p. Since the currents are proportional to the square roots of the dynamometer readings it follows that the accuracy is again low at low loads.

The two-ammeter method serves for quick ratio tests, such as are frequently required when checking ammeter transformers on site; the method is useless for wattmeter transformers since it is not possible to obtain by its use any measurement of the phase-angle β.

2. Two dynamometer methods.

Two dynamometers used in conjunction with a phase shifting device/

* L.T.Robinson, "Electrical Measurements on circuits requiring current and potential transformers," Trans.Amer.I.E.E., vol.28, pp.1005-1039, Discussion pp.1040-1052, 1910.



Simple Two Dynamometer Method

FIG. 15.

device enable the ratio and phase-angle to be readily determined; a large number of methods have been described having this basic principle. Referring to Fig.15a, the primary of the transformer is connected to a source of a.c. in series with an ammeter A_p and the current coils of a dynamometer or wattmeter D_p of suitable range and sensitivity; the secondary is closed through an ammeter A_s and the current coils of a second dynamometer or wattmeter D_s. The voltage coils of D_p and D_s are joined in parallel across an auxiliary source of supply the phase of which relative to the source feeding the primary circuit can be adjusted. The auxiliary supply can be taken from a phase shifting transformer excited from the source which supplies the primary; or the primary and auxiliary sources may be two similar coupled alternators with means for effecting relative phase relationships. The ammeters A_p, A_s are convenient for setting the load to a desired value but are not essential; A_p may be retained, but any desired burden can be substituted for A_s.

The process* is very simple and is illustrated by the vector diagrams of Fig.15 b and c. Assuming that the voltage coils of the two dynamometers are of negligible reactance so that/

* L.T.Robinson, Trans.Amer.I.E.E., vol.25, pp.727-734, 1907 and vol.28, pp.1005-1039, 1910 uses the process only for the determination of angle, finding the ratio by the shunted dynamometer method of the preceding section. The complete process here described has been used by Kartak, loc.cit., 1920, Spilsbury, loc.cit., 1920, A.Barbagelata, L'Elettrotecnica vol.8, pp.165-175, 1921 and Silsbee loc.cit., 1924. A.G.L. McNaughton, "The current transformer," Journal I.E.E., vol.53, pp.269-271, 1915 uses the above process to find the angle β, but obtains the ratio from the ammeter readings.

that the currents in them are in phase with the auxiliary voltage E applied by the phase shifter and indicated by the voltmeter V ,adjust the phase shifter until the reading of D_p is a maximum. Then E and I_p are in phase, and the readings of D_p and D_s will be, as can be verified from Fig. 15b,

$$W_p = EI_p \text{ watts,}$$
$$W_s = EI_s \cos(\pi - \beta) = -EI_s \cos\beta \text{ watts.}$$

Since β is small the numerical value of W_s is nearly EI_s so that

$$K_c \doteqdot \frac{W_p}{W_s}.$$

Now adjust the phase shifter until D_p reads zero, then E and I_p are in quadrature as shown in Fig.15c and the new reading of D_s is

$$W_s' = EI_s \cos\left(\frac{3\pi}{2} - \beta\right) = -EI_s \sin\beta,$$

$$\text{or} \quad \beta \doteqdot \sin\beta \doteqdot \frac{W_s'}{W_s}; \text{ or accurately } \tan\beta = \frac{W_s'}{W_s}$$

The method is simple and quick, and an accuracy of 0.2% can be obtained at full load. The sensitivity falls off only in proportion to the load, so that as a method for measuring ratio it is to be preferred to the shunted dynamometer method in which the sensitiveness decreases in proportion to the square of the current. The method checks the polarity of the transformer terminals, which is an advantage in practice.

The/

The disadvantages of the method are three in number:- (i) The secondary burden imposed by the dynamometer D_s is considerable and is about 10 volt-amperes; (ii) The range of primary current is limited to about 200 amperes, which is the greatest current for which dynamometer instruments can be easily constructed without the use of auxiliary transformers. In addition, a number of primary dynamometers will be required to retain sensitivity over the wide range of maximum primary currents found in practical transformers; (iii) The reactance of the volt coils of D_p and D_s will cause an error, neglected in the above simple theory.

The reactance error is easily determined. Let θ_p and θ_s be the angles of phase displacement between the currents in the voltage circuits of D_p and D_s and the common applied voltage E. Then with D_p reading a maximum it is easy to show that the readings in watts of D_p and D_s will be $W_p = EI_p \cos\theta_p$, and $W_s = EI_s \cos\theta_s . \cos(\pi - \beta + \theta_p - \theta_s)$ $= - EI_s \cos\theta_s . \cos(\beta + \theta_s - \theta_p)$, so that $W_p/W_s = [I_p \cos\theta_p]/[I_s \cos\theta_s . \cos(\beta + \theta_s - \theta_p)] \doteq I_p/I_s \doteq K_c$, because $\theta_s, \theta_p, \beta$ are all small and of similar magnitude. Now adjust the phase of E until D_p reads zero; then if E has been made to lead I_p by $\frac{\pi}{2} + \theta_p$ this result will be secured and the reading of D_s becomes $W_s' = EI_s \cos\theta_s . \cos(\frac{3\pi}{2} - \beta + \theta_p - \theta_s)$ $= - EI_s \cos\theta_s . \sin(\beta + \theta_s - \theta_p)$, whence $W_s'/W_s = \tan(\beta + \theta_s - \theta_p)$. Hence reactance in the volt coils produces no appreciable error in K_c but may have a considerable effect on the value of β. The error can be made zero, i.e., $W_s'/W_s = \tan\beta$ if $\theta_s = \theta_p$ or if $\theta_s = \theta_p = 0$. That is, if the volt circuits have equal time-constants, and hence equal phase displacements the error in β will be zero. It will also be zero if the reactance of both voltage coils be annulled by any of the well-known methods, e.g., the inclusion of a properly chosen shunted condenser in each volt circuit. The error can also be avoided, as suggested independently by Moore* and by Barbagelata, if the volt coils are joined in series instead of in parallel and if the auxiliary current, I, be measured instead of the auxiliary voltage.

It is possible to determine β by adjusting the phase shifter/

* A.E.Moore, Journal I.E.E., vol.51, pp.346-347, 1913; A. Barbagelata, loc.cit., 1924.

shifter so that D_p and D_s are successively brought to zero;* β is then equal to the angle through which the phase has been rotated to obtain the two zero readings. Since β is so small - never more than 3° - this necessitates some magnifying device for the accurate reading of the angle upon the scale of the phase shifter; this may be effected by optical means. The necessity for magnifying the angle to enable it to be accurately measured may be avoided by using a process described by Makower and Wust.† Neglecting volt coil reactance effects let E be adjusted to lead on I_p by an angle ϕ ; then the readings in watts of D_p and D_s will be

$$W_p = EI_p \cos\phi$$

and $$W_s = EI_s \cos(\pi-\beta+\phi) = -EI_s \cos(\phi-\beta).$$

Then, $$\frac{W_s}{W_p} = -\frac{I_s}{I_p}\cdot\frac{\cos(\phi-\beta)}{\cos\phi} = -\frac{1}{K_c}\left[\cos\beta+\sin\beta.\tan\phi\right] = a \quad \text{say;}$$

taking various values of ϕ let a be determined and a curve showing a as a function of ϕ be plotted. Then if a_1, a_2 be values of a from the curve corresponding with angles ϕ_1, ϕ_2 it is easy to show that

$$\tan\beta = \frac{a_1 - a_2}{a_2\tan\phi_1 - a_1\tan\phi_2},$$

from which β is found. Putting this value of β in the

expression/

* A.E.Moore, loc.cit., 1913.

† A.J.Makower, "Measurement of phase difference," Elecn., vol.58,p.695,1907; A.J.Makower and A.Wust, "Phase lag in current transformers," Elecn.,vol.79,pp.581-582,p.671, 785, 1917.

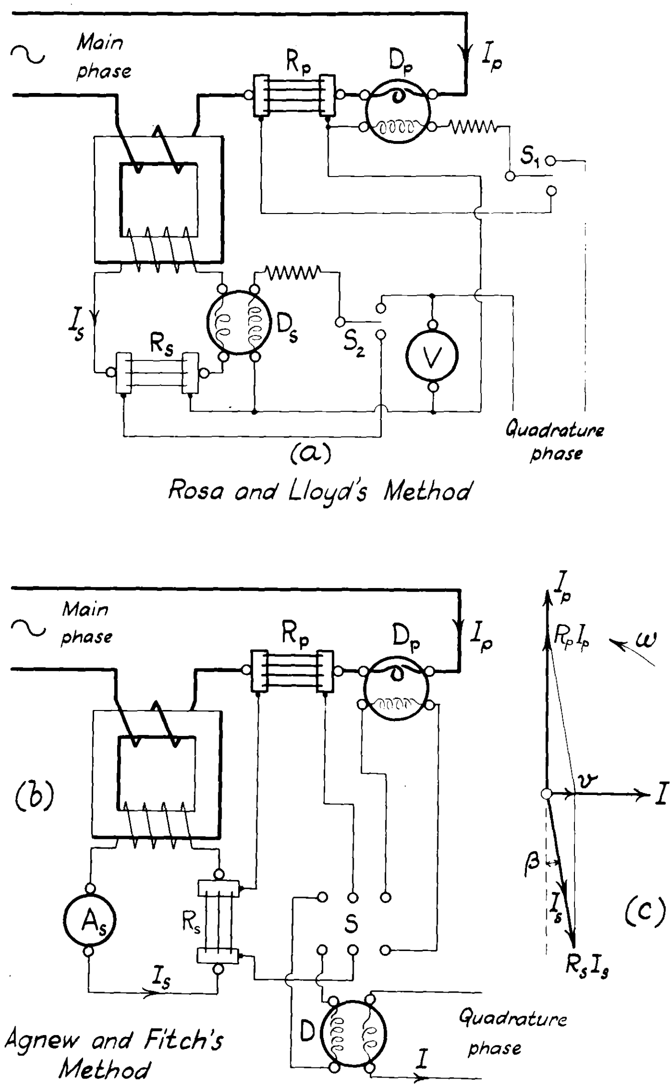

Main
phase
R_p
D_p
I_p
S_1
I_s
R_s
D_s
S_2
V
Quadrature
phase
(a)
Rosa and Lloyd's Method
Main
phase
R_p
D_p
I_p
(b)
A_s
R_s
I_s
S
D
I
Quadrature
phase
Agnew and Fitch's
Method
I_p
R_pI_p
ω
v
I
β
I_s
R_sI_s
(c)

FIG. 16.

expression for a_1 enables K_c to be calculated.

A number of important modifications of the two dynamometer method, in which a polyphase source of supply takes the plače of the phase-shifter, have been used and must now be considered.

Of the two-phase methods that of Rosa and Lloyd* is shown in Fig.16a. The dynamometers D_p, D_s are of the reflecting type in which the reactance of the voltage circuits has been compensated. The ratio of the transformer is found by throwing down the switches S_1 and S_2, and using the dynamometers to measure the primary and secondary currents, just as in Fig.14b. To find the phase-angle β, S_1 and S_2 are thrown up thus connecting the voltage coils of the two dynamometers in series across the second phase, which is in quadrature with the main phase supplying the transformer. The volt coils thus carry current in quadrature with I_p and D_p will cease to read; in consequence of the angle β, D_s will give a reading

$$d = k I_s I \sin\beta = k I_s \frac{E}{r_s} \sin\beta,$$

where k is the dynamometer constant, I the current in the volt coils, r_s the total resistance of the voltage circuit of D_s, and E the voltage across that circuit, read upon the voltmeter V. If now the switch S_2 be thrown down and the/

*- E.B.Rosa and M.G.Lloyd, "The determination of the ratio of transformation and of the phase relations in transformers," Bull.Bur.Stds., vol.6, pp.1-30, 1910.

the resistance of the volt coil circuit of D_s, now a shunted dynamometer, be adjusted to a value r_s' so that the deflection is again d,

$$d = k\, I_s . I_s \frac{R_s}{R_s + r_s'}$$

so that $\sin\beta = \dfrac{R_s I_s}{R_s + r_s'} . \dfrac{r_s}{E}$.

The defect of the method is that the dynamometers used had a fairly high resistance so that the transformer is working under unpractical conditions. This defect is overcome by the method of Agnew and Fitch,* shown in Fig.16b. With the aid of this method these experimenters made one of the earliest extensive investigations of the properties of current transformers. The four-terminal resistances R_p and R_s are chosen so that each gives a volt drop of 0.1 to 0.4 volts, i.e., R_s/R_p is made about equal to the nominal ratio of the transformer. R_s must be adjustable and may consist of a four-terminal resistance shunted by a variable and much larger resistance, or alternatively may be a four-terminal slider resistance. The burden imposed by R_s is from 0.5 to 2 volt-amperes.

R_p and R_s are connected together as shown so that the drops of voltage in them are in opposition; with the switch to/

* P.G.Agnew and T.T.Fitch, "The determination of the constants of instrument transformers," Bull.Bur.Stds., vol.6, pp. 281-299, 1910. P.G.Agnew, "A study of the current transformer with particular reference to iron loss," Bull.Bur. Stds., vol.7,pp.423-474, 1911. For another two phase method see Barbagelata, loc.cit.ante, 1921.

to the right R_s is adjusted until D_p reads zero. The resultant voltage, v , compounded of $R_p I_p$ and $R_s I_s$ in the way shown in Fig.16c is then in quadrature with I_p. The switch is then turned to the left and the reading of the dynamometer D taken. The current I in the fixed coils of this instrument is supplied from a source in quadrature with I_p and hence the reading of D is a measure of v ; the dynamometer may be a wattmeter calibrated in watts or the scale may be marked in volts by applying known voltages to the moving coils while I is maintained constant in the fixed coils. From the simple geometry of the vector diagram,

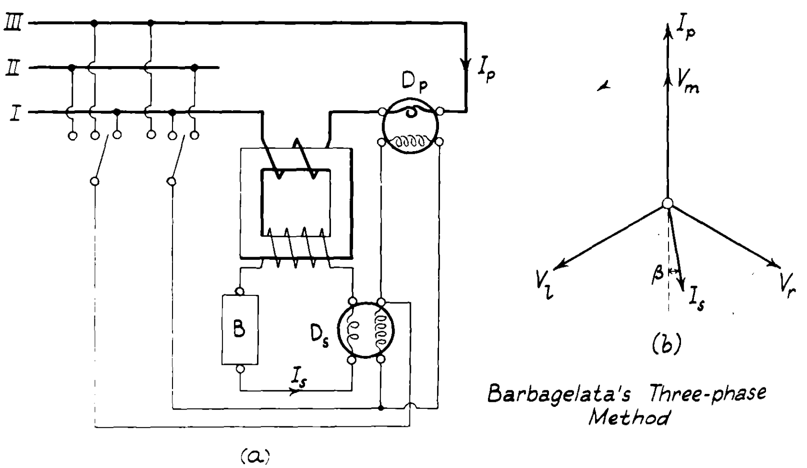
$$R_s I_s \cos\beta = R_p I_p$$

whence $$K_c = \frac{I_p}{I_s} = \frac{R_s}{R_p}\cos\beta \doteqdot \frac{R_s}{R_p}.$$

Also $$v = R_p I_p \tan\beta = R_s I_s \sin\beta$$

whence β can be found.

The method of Agnew and Fitch is capable of considerable precision; these experimenters used reflecting dynamometers and were able to find K_c to 0.005% and β to the nearest minute. The sensitivity, however, falls off as the square of the current and so two-range dynamometers were used to maintain this degree of precision over a wide range of currents. The method is subject to certain errors, chiefly in the determination of β , arising from the reactance of the voltage coils of D and the residual reactance of R_p and/



Barbagelata's Three-phase Method

FIG. 17

and R_s. These are investigated at length in the original paper, where it is shown that the effect of the volt coil reactance can be made negligible by the adoption of the usual methods of compensation. For the residuals, it is shown that their effects on β are opposite in sign; hence if R_p and R_s are <u>both</u> slightly inductive (or capacitive), residual errors will vanish when they have equal time-constants. The ammeter A serves for adjustment of the desired secondary load, but can be replaced by any required burden.

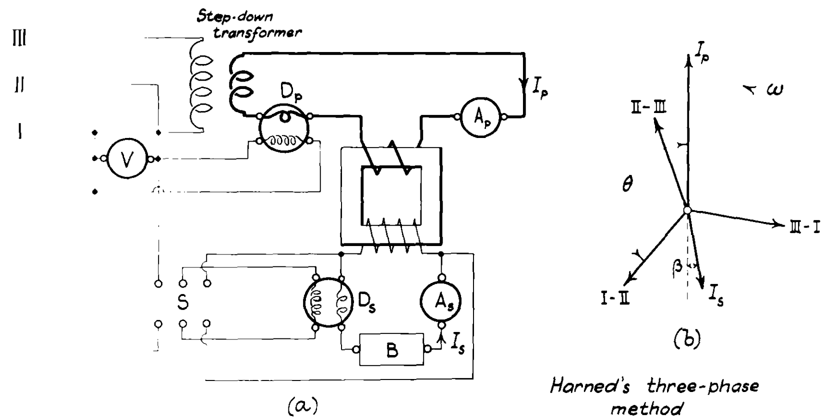
Turning now to three-phase methods, the simplest is that due to Barbagelata,* shown in Fig.17a. Assuming the volt coils of D_p and D_s to be compensated for reactance, throwing the switch upon the middle, right and left hand contacts successively will apply voltages between lines III and I, I and II, II and III to the voltage circuits in parallel. Let these three voltages be V_m, V_r and V_l in Fig.17b, each of equal amplitude V and successively displaced in phase by 120°. Take readings of the two dynamometers, in watts, with the switch in the three positions and let a, b and c be the ratio of the reading of D_p to that of D_s. Then noting that I_p is nearly in phase with V_m

$$a = \frac{V_m I_p}{V_m I_s \cos(\pi-\beta)} = \frac{I_p}{-I_s \cos\beta};$$

$$b = \frac{V_r I_p \cos\frac{4\pi}{3}}{V_r I_s \cos(\frac{\pi}{3}-\beta)} = \frac{-I_p \cos\frac{\pi}{3}}{I_s \cos(\frac{\pi}{3}-\beta)};$$

$$c = \frac{V_l I_p \cos\frac{2\pi}{3}}{V_l I_s \cos(2\pi-\frac{\pi}{3}-\beta)} = \frac{-I_p \cos\frac{\pi}{3}}{I_s \cos(\frac{\pi}{3}+\beta)}.$$

* A.Barbagelata, <u>loc.cit.ante.</u>, 1921; a two-phase method is also described.



Harned's three-phase method

FIG. 18.

Remembering that β is a small angle it is easy to show that

$$K_c = \frac{I_p}{I_s} \doteq a \quad \text{numerically}$$

and $$\sin\beta \doteq \frac{b-c}{2\sqrt{3}\,a} \doteq \beta.$$

A second method, due to Harned,* is shown in Fig.18a. This method is primarily intended for testing of transformers on site by means of portable instruments and an ordinary three-phase supply. D_p and D_s are wattmeters, the former being excited from the voltage between lines I and II. D_s is first excited across the secondary of the transformer by putting S to the right, enabling the secondary volt-amperes to be adjusted to a desired value. With S to the left, both dynamometers are excited from lines I and II and will give readings $W_p = EI_p\cos\theta$, $W_s = EI_s\cos(\theta-\pi-\beta)$; eliminating θ the unknown angle between the voltage E indicated by the voltmeter V and I_p ,

$$\beta = \pi + \arccos\frac{W_p}{EI_p} - \arccos\frac{W_s}{EI_s}.$$

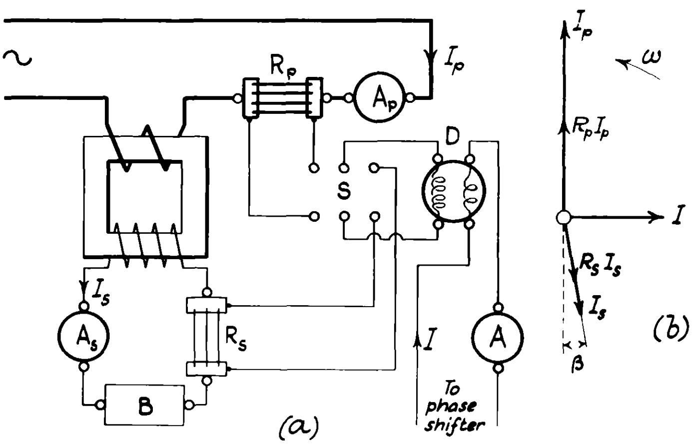
The ratio is found from the ammeter readings and is subject to the usual errors.

3. <u>Single dynamometer methods</u>.

The use of a single dynamometer to measure the characteristics of a transformer appears to have been used at about the same time by Robinson in America and Drysdale in England.

The/

* M.L.Harned, "Operating characteristics of current transformers," <u>Elec.World</u>, vol.67,pp.869-872, 1916.



Simple Single Dynamometer Method

FIG. 19.

The method used by Robinson,[*] in its most complete form is shown in Fig.19a. The ratio is obtained by adjusting the phase shifter to give a maximum reading on the dynamometer D when S is put to the right and to the left successively, the reading of A being kept constant at a value I. These two maximum readings are a measure of I_s and I_p respectively. It is best to choose R_p and R_s to give approximately equal drops of voltage $R_p I_p$ and $R_s I_s$; then if W_s and W_p are the readings in watts $W_s = R_s I_s I$ and $W_p = R_p I_p I$, giving $K_c = \frac{I_p}{I_s} = \frac{R_s}{R_p} \cdot \frac{W_p}{W_s}$, neglecting the shunting effect of the volt circuit of D. To get the angle, S is thrown to the left and the phase shifter adjusted until D reads zero; then I is in quadrature with I_p , as shown in Fig. 19b. Then with S to the right the reading W of D is noted,

$$W = R_s I_s \frac{r}{r+R_s} \cdot I \cos\left(\frac{3\pi}{2} + \beta\right) = \frac{r R_s}{r+R_s} \cdot I I_s \sin\beta$$

where r is the resistance of the voltage circuit of D , assumed of very low or of compensated reactance. Then,

$$\sin\beta = \frac{r+R_s}{r R_s} \cdot \frac{W}{I I_s} \doteq \frac{W}{R_s I_s I} \doteq \beta \doteq \frac{W}{W_s}$$

if r is large in comparison with R_s. The sensitivity is proportional to the current.

Rosa and Lloyd[†] have used the method in a slightly different way. The ratio is found from the ammeter readings. The/

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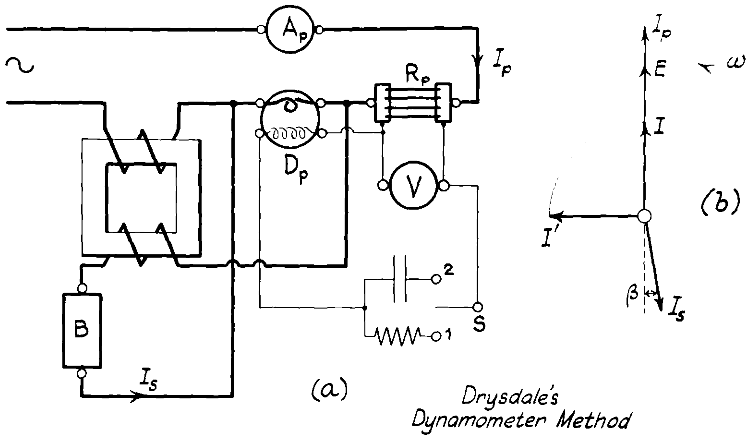
[*] L.T.Robinson, Trans.Amer.I.E.E., vol.25, pp.727-734, 1907.

[†] E.B.Rosa and M.G.Lloyd, loc.cit., 1910.



A_p
I_p
R_p
D_p
V
B
I_s
2
1
S
(a)
I_p
E
ω
I
(b)
I'
β
I_s
Drysdale's Dynamometer Method

FIG. 20.

The phase-angle is determined by adjusting the phase shifter first to make D read zero when connected to R_p and then when connected to R_s. The change in the phase-shifter setting is then the value of β, and can be read to 0.1° or less with the aid of a vernier or optical magnification. The method is very quick and quite accurate enough for works testing. The burden imposed by R_s is slight; in a 125/5 ampere transformer R_p was 0.001 and R_s was 0.025 ohm, so that the burden due to R_s was 0.625 volt-ampere.

Drysdale's method* is somewhat different from the preceding and is shown in Fig.20a. The method was originally devised to test transformers of 1/1 ratio and is not directly applicable to other cases. The primary and secondary currents of the transformer are caused to pass, approximately in opposition, through the fixed or current coil of the dynamometer D_p. Alternatively a dynamometer with two current coils may be used, the primary current passing through one and the secondary current through the other. By choosing the numbers of turns in these coils so that their ampere-turns are approximately equal, ratios other than 1/1 may be dealt with. Confining attention to a unity ratio transformer as shown in the diagram, the vector relationships will be

those/

* C.V.Drysdale, "The measurement of phase differences," Elecn., vol.57,pp.726-728, 783-784, 1906; "Some measurements on phase displacement in resistances and transformers," Elecn., vol.58,pp.160-161, 199-201, 1907; "The use of shunts and transformers with alternate current measuring instruments," Phil.Mag., 6th series, vol.16,pp.136-153, 1908.

those drawn in Fig.20b, and the procedure will be as follows. If E is the voltage indicated by the voltmeter V, E will be in phase with the primary current I_p; with S on contact 1 the current I in the volt coils of D_p will, neglecting their reactance,* be in phase with I_p and the dynamometer will read W watts where

$$W = EI_p + EI_s \cos(\pi - \beta) = EI_p - EI_s \cos\beta$$

With S on contact 2 a condenser equal in reactance to the resistance just removed is inserted in the volt circuit of D_p, the current I' leading by $\pi/2$ on E and being equal to I in magnitude. The insertion of the condenser has the same effect on D_p as retaining the non-reactive volt circuit and advancing the phase of E by $\pi/2$. The new reading of D_p being W',

$$W' = EI_s \cos\left(\frac{3\pi}{2} - \beta\right) = -EI_s \sin\beta$$

From these equations

$$K_c = \frac{I_p}{I_s} = \frac{EI_p \cos\beta}{EI_p - W} \doteq \frac{EI_p}{EI_p - W}$$

$$\text{and} \quad \tan\beta = \frac{W'}{W - EI_p}$$

Instead of using a voltmeter E can be determined from the observed value of I_p and the value of R_p.

A method, resembling in some respects that of Agnew and Fitch (see Fig.16b), has been used by Barbagelata† and is/

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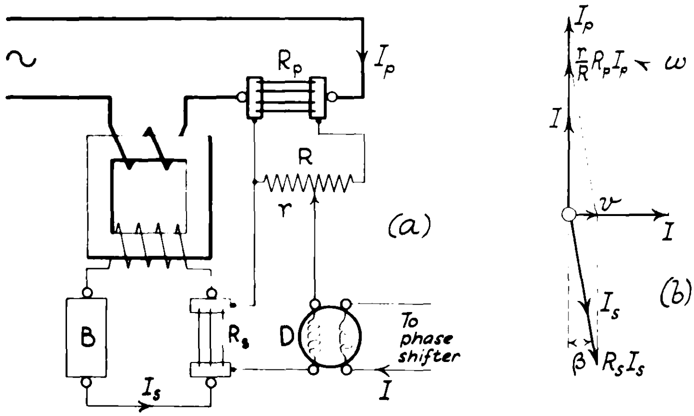
* For a discussion of the effect of volt coil reactance and an alternative theory of the method see C.V.Drysdale and A.C.Jolley, "Electrical Measuring Instruments, vol.2, pp.293-294, 1924.

† A.Barbagelata, loc.cit., 1921.



Barbagelata's Single Dynamometer Method

FIG. 21.

is illustrated in Fig.21a. The auxiliary current I is first set in phase with I_p and r is adjusted until the reading of D is zero. The voltage v applied to the volt coil of D is the resultant of $\frac{r}{R} R_p I_p$ and $R_s I_s$, and is normal to I and I_p . If the phase of I be changed by 90°, so that v and I are in phase, the reading of D will be vI or

$$W = R_s I_s \sin\beta . I \text{ watts}$$

From the Fig.21b. $R_s I_s \cos\beta = \frac{r}{R} R_p I_p$,

$$\text{whence} \qquad K_c = \frac{I_p}{I_s} = \frac{R}{r} \cdot \frac{R_s}{R_p} \cos\beta \doteqdot \frac{R}{r} \cdot \frac{R_s}{R_p},$$

$$\text{and} \qquad \sin\beta = \frac{W}{R_s I_s I}$$

$$\text{or} \qquad \tan\beta = \frac{W}{I} \cdot \frac{r}{R} R_p I_p$$

Hence I_p (or I_s) and I must be measured. The method can also be used by a null process as described in Section 1 of Chapter IV.

Palm* in a long and detailed paper has described a method similar to that of Barbagelata but using a more complicated series of observations with a view to making allowance for the shunting effect of the voltage circuit and other slight sources of error.

A/

* A.Palm, "Prüfung von Messtransformatoren mit dem Spiegel-Elektrodynamometer," Zeits.f.Inst., vol.34,pp.281-290, 1914.

A method of low precision has been described by Dawes,[*] making use of ordinary pointer instruments, for measuring the phase-angle of transformers on site. In principle the readings of a wattmeter are taken when its volt coils are excited from a constant voltage source (i) when a desired current is passed directly through the current coils and (ii) when the desired current is obtained from the secondary of the transformer that is to be tested. Any difference between the readings is due to the phase-displacement introduced by the transformer. As in most difference methods great care is necessary if accurate results are to be obtained.

4. Watt-hour meter method.

The method now to be briefly discussed was originally devised[†] to enable a current transformer to be standardised without the use of laboratory apparatus. The transformer in question had subdivided primary and secondary windings, the coils of which were grouped to make the nominal ratio unity, and the transformer was tested with the coils so connected. It is well-known that the ratio and phase-angle of/

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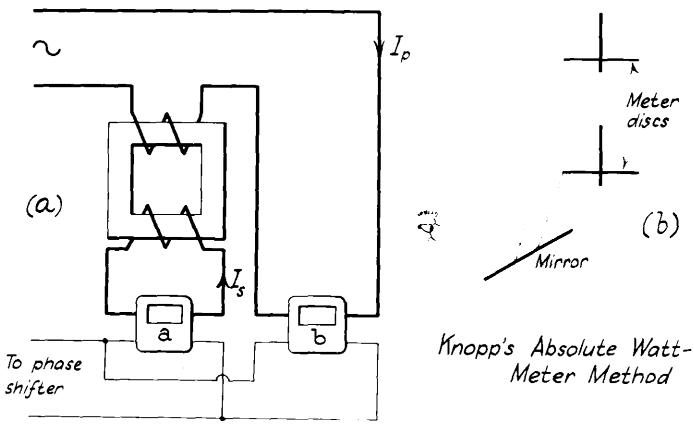
* C.L.Dawes, "The phase angle of current transformers," Proc.Amer.I.E.E., vol.34, pp.927-940, 1915.

† O.Knopp, "The commercial standardization of instrument transformers," Elec.World, vol. 67, pp. 92-93, 1916.



Knopp's Absolute Watt-hour Meter Method

FIG. 22.

of such multirange transformers is practically independent of the grouping of the coils, provided that care is taken in the construction of the transformer; hence they may be regrouped for use and the test results will apply very closely.

The principle of the method* is very simple. Two a.c. meters are adjusted so that one runs about 10% faster than the other; the speed ratio of the meters is found when the current elements are supplied in series with normal current and the volt coils in parallel at rated voltage. The current element of one meter is now put in the primary circuit and that of the other in the secondary circuit of the transformer that is to be tested, the voltage circuits being supplied from a phase shifter, as shown in Fig.22a. With the phase shifter adjusted so that the meters are working at an apparent power-factor of about unity the speed ratio is again determined. The change in speed ratio of the meters is a measure of the ratio error of the transformer. The speed ratio is again found when the apparent power-factor has been adjusted to about 0.5 and from these observations β can be calculated. Observations of speed ratio are most easily made by mounting the meters one above the other and viewing both discs simultaneously in a mirror, as shown in Fig.22b; the speed ratio is then easily deduced/

* See also H.M.Crothers, "Field testing of instrument transformers," Elec.World, vol.75,pp.319-320,1920; J.A.Kartak, loc.cit.ante., 1920.

deduced from the observed coincidences of the marked spots on the meter discs.

The theory of the method is given in Section 4 of Chapter V as a special case of Agnew's watt-hour meter method of comparing two transformers; to this discussion the reader is directed.

5. Electrometer methods.

It is now necessary to notice certain methods, closely related to those already described, in which the measuring instrument is a reflecting quadrant electrometer. These methods have assumed a considerable practical importance since they have been established, first at the Reichsanstalt and later at the National Physical Laboratory as the standard process for absolute tests on instrument transformers. Electrostatic methods of measuring alternating quantities have been developed to a high degree of perfection in both these institutions and the advantages of electrostatic instruments for such a purpose are too well known to be repeated here. For the present purpose it is sufficient to look into the question of electrometer methods of testing transformers in a broad general way and with little attention to detail; the provision of a suitable electrometer and the technique of its satisfactory use are matters of a highly specialised nature, so much so, indeed, that it is hardly likely that electrometer methods would be set up in any other place than a national laboratory. The following discussion will be brief/

brief, and the reader desirous of further detail will find the papers referred to, and the bibliographies contained therein, of considerable value.

Electrostatic measurements of small phase displacements were made by Drysdale* in 1907 but it was not until Schultze and Orlich developed and modified the Kelvin electrometer then in use that such measurements could be made with comparative ease. Schultze† in the same year described a reflecting electrometer specially designed for a.c. testing in the Reichsanstalt; the N.P.L. instrument, based on Schultze's design, was described‡ in 1913. These electrometers were made to overcome the numerous difficulties inherent in the Kelvin instrument and to possess greater electric stability, ease of operation, and sensitiveness.

All electrometer methods for current transformer testing are based on the principle of using an electrometer instead of a dynamometer to compare the voltages over two four-terminal resistances, one in the primary and one in the secondary circuit. In the Reichsanstalt method due to Orlich,** shown in simplified form in Fig.23a, R_p

and/

* C.V.Drysdale, "Some measurements on phase displacement in resistances and transformers," Elecn.,vol.58,pp.160-161, 199-201,1907.

† H.Schultze, Zeits.f.Inst., vol.27, p.65, 1907.

‡ C.C.Paterson, E.H.Rayner, A.Kinnes, Journal I.E.E.,vol.51, pp.294-330, 1913.

**E.Orlich, "Uber die Anwendung des Quadranten-elektrometers zu Wechselstrommessungen," Elekt.Zeits., vol.30, pp.435-439, 466-470, 1909.

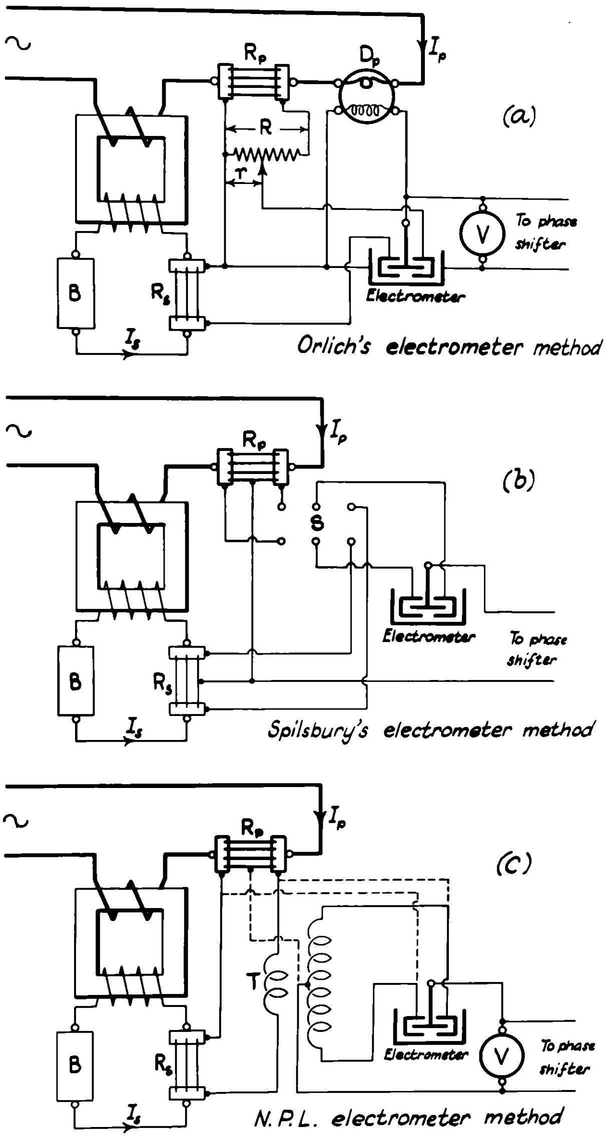

~
R_p
D_p
I_p
(a)
R
r
V
To phase shifter
B
R_s
Electrometer
I_s
Orlich's electrometer method
~
R_p
I_p
(b)
S
B
R_s
Electrometer
To phase shifter
I_s
Spilsbury's electrometer method
~
R_p
I_p
(c)
T
B
R_s
Electrometer
V
To phase shifter
I_s
N.P.L. electrometer method

FIG. 23.

and R_s are low resistances (for a 100/5 ampere transformer R_p is 0.01 ohm and R_s 0.1 ohm) the former shunted by a resistance R of 100 ohms. The drop of voltage over r and that over R_s are in approximate opposition and their resultant is applied to the quadrants of the electrometer. An auxiliary voltage of 50 to 100 volts is maintained between the needle and case of the electrometer by means of a phase shifting device. The dynamometer D_p facilitates phase settings of the voltage E, indicated by the voltmeter V, relative to the primary current. E is first set in phase with I_p and then r is adjusted until the electrometer reads zero; then the resultant voltage applied to the quadrants is in quadrature with E and

$$R_s I_s \cos\beta = \frac{r R_p}{R+R_p} I_p$$

or $$K_c = \frac{I_p}{I_s} = \frac{R_s}{r}\left(1+\frac{R}{R_p}\right)\cos\beta \doteq \frac{R_s R}{R_p r}\left(1+\frac{R_p}{R}\right)$$

The phase of E is then changed by 90° and the deflection of the electrometer observed; this deflection is proportional to β, the phase-angle, very nearly. The method is the electrostatic parallel to Barbagelata's single dynamometer method of Fig.21. A precision of 1 part in 10000 in ratio and 0.1 minute of angle is claimed for the method.

Spilsbury* has described the method shown in Fig.23b

which/

* R.S.J.Spilsbury, Beama, vol.6, pp.505-513, 1920.

which should be compared with the single dynamometer method of Fig.19a. A p.d. of about 100 volts is maintained between the needle and the mid points of R_p and R_s. With the switch S to the left the phase shifter is adjusted to give a maximum reading the electrometer; similarly the reading is observed with S to the right. These readings d_p, d_s are proportional to $R_p I_p$ and $R_s I_s$ so that

$$K_c = \frac{d_p}{d_s} \cdot \frac{R_s}{R_p}$$

The phase of the auxiliary supply is regulated until when S is to the left the electrometer reads zero; then I_p and the voltage on the needle are in quadrature. If S is now thrown to the right, the resulting reading is nearly proportional to β, exactly as in the corresponding dynamometer method. To get sufficient sensitiveness the p.d. on the quadrants must have a reasonably high value, necessitating a fairly large secondary burden due to R_s, usually about 10 volt-amperes.

The disadvantage of the preceding method is overcome in the method* used at the N.P.L. by the connections shown in Fig.23c. Here $R_p I_p$ and $R_s I_s$ are made about equal and their small vector sum is impressed on the quadrants through a 100 to 1 step-up transformer T. A p.d. of 100 volts is applied between the needle and the mid-point of the secondary of T. Alternatively, operation of a switch, not shown in the/

* R.S.J.Spilsbury, "A new method of testing current transformers," Elecn.,vol. 86, pp.296-297, 1921.

the sketch, alters the connections between the resistances so that the drop across R_p is impressed on the quadrants while the auxiliary voltage is applied between the needle and the mid point of R_p ; connections to R_s and T are removed. This alternative circuit is indicated by the dotted lines. With the dotted connections the phase shifter is adjusted to make the electrometer read zero; reverting to the full-line connections the electrometer will give a deflection proportional nearly to β . Again with the dotted connections the phase shifter is adjusted by 90°, until the electrometer gives a maximum reading proportional to I_p . With the full-line connections a third reading is taken. Then clearly, if d_1, d_2, d_3 be the readings

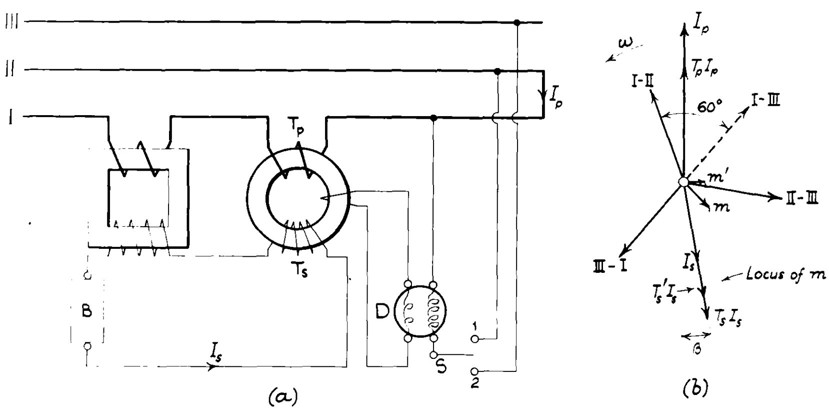
$$d_1 = 100 KE R_s I_s \sin\beta,$$
$$d_2 = KER_p I_p,$$
$$d_3 = 100 KE(R_s I_s \cos\beta - R_p I_p),$$

where E is the auxiliary voltage, K the electrometer constant and 100 the transformation ratio of T . From these

$$K_c = \frac{I_p}{I_s} = \frac{R_s}{R_p} \cdot \frac{100\, d_2}{d_3 - 100 d_2} \cos\beta \doteq \frac{R_s}{R_p} \cdot \frac{100\, d_2}{d_3 - 100 d_2}$$

$$\tan\beta = \frac{d_1}{d_3 - 100 d_2} \doteq \beta$$

R_s is about 0.4 ohm. (burden 2 volt amperes) and causes 280 cm. deflection for 1% error in ratio or for 30 minutes in angle; ratio can be found within 0.1% and angle to nearest/



Baker's test ring method

FIG. 24.

nearest minute. Possible sources of error are (i) phase and ratio imperfections in T ; (ii) magnetising current taken by T upsetting drops in R_p and R_s ; (iii) effect of stray fields on T ; (iv) residual inductance of R_p and R_s . These factors are shown, in general, to cause negligible errors.

6. <u>Baker's test ring</u>.

A method described by Baker[x] utilises a principle quite different from those hitherto discussed. In this the primary and secondary currents are compared by passing them through separate windings upon a laminated iron ring and finding the resulting magnetomotive force that they set up. Referring to Fig.24a. the m.m.f. of the primary and secondary currents act in approximate opposition round the ring and the ratio of the turns T_p and T_s in the windings can be so chosen that their ampere-turns are nearly equal; a small flux is set up in the ring by the resultant m.m.f. and this is linked with a tertiary winding connected to one of the coils of a dynamometer D . The other coil of D can be excited at will by the voltages between lines I and II or I and III of a three-phase supply. Referring to the vector diagram, Fig.24b, the voltages between the pairs of lines are shown in relation to the primary and secondary currents of the transformer. The resultant ampere-turns/

x H.S.Baker, "Current ratio and phase angle test of series transformers," <u>Elec.World</u>, vol.57, pp.234-235, 1911; "Current transformer ratio and phase error by test ring method," <u>Proc.Amer.I.E.E.</u>, vol.37, pp.1173-1183, 1918. See also F.B.Silsbee, <u>loc.cit.ante.</u>, 1924.

turns on the ring will be m, responsible for the flux in the ring and the voltage applied to the dynamometer coil. A reading of the dynamometer is taken with S first on 1 and then on 2; these readings are proportional to the components of m in the direction of the voltages I-II and I-III respectively. These two readings are repeated for say two other values of T_s. Since $T_p I_p$ is constant and $T_s I_s$ is in the direction of I_s it follows that the locus of m is a line parallel to I_s through the extremity of $T_p I_p$. From the observations the position of m can be set out on a sheet of paper for the three chosen values of T_s and the locus found. From this locus it is easy to interpolate the value of T_s, say T_s', which would make m perpendicular to I_p. Then

$$T_s' I_s \cos\beta = T_p I_p$$
$$m' = T_p I_p \tan\beta = T_s' I_s \sin\beta$$

where m' is the measured length of m when normal to I_p; thus

$$K_c = \frac{I_p}{I_s} = \frac{T_s'}{T_p}\cos\beta \doteq \frac{T_s'}{T_p}$$
$$\tan\beta = \frac{m'}{T_p I_p} \doteq \beta, \quad \text{or} \quad \sin\beta = \frac{m'}{T_s' I_s} \doteq \beta.$$

In Baker's apparatus the ring is built up of laminations 8" inside diameter to a section of 2" x 2"; the flux density in it is about 100 lines per square inch. The wattmeter consists of 196 turns of No. 16 wire uniformly wound on the ring. The secondary coil coil is made of No. 8 wire and contains 4 coils of 40 turns, 2 of/

of 20 turns, and 1 of 10 turns, each uniformly distributed over the ring; in addition a further distributed coil of 20 turns tapped at every turn is provided. Any desired number of secondary turns can be arranged. The primary consists of a number of U shaped copper loops spaced out uniformly round the ring and arranged for series, parallel or series-parallel grouping. The wattmeter is preferably of the zero pattern and the coil connected to the ring should have low resistance; this serves to minimise the burden imposed on the test transformer by keeping the flux in the ring to a low value in consequence of the low e.m.f. required in the tertiary winding. The method gives results in excellent agreement with other methods, it is very flexible, and is easily applicable to testing of transformers on site.

7. Use of Phase meter.

Among the less usual methods of measuring the phase-angle of a current transformer may be mentioned the use of a phase meter. For this purpose the ordinary type, with three equal voltage coils mounted on a common spindle and supplied from a three-phase network while the fixed current coil is connected in one line, is not suitable owing to its low sensitiveness to small angular displacements of phase. To attain a sufficient magnification Gifford* has made a phase meter in which a high magnification is secured by winding the voltage coils with

unequal/

* E.D.Gifford, "A method of determining the phase-angle of current and potential transformers," Elecn., vol.75, pp. 166-167, 1915.

unequal numbers of turns so that they set up an elliptic instead of a circular rotating field. The current coil reacts with the major axis of the elliptic field and produces a torque which, by making the field of suitable ratio of major to minor strength, can be made large for small angles. The current coil is switched successively from primary to secondary circuit of the transformer and measures directly the phase difference between the currents therein. Though capable of good results and of speed in making a test the instrument has not come into regular use.

8. Use of Oscillograph.

It remains now to mention the use of the oscillograph for finding the ratio and phase-angle of a transformer by taking simultaneous wave-form records of primary and secondary currents. Some experimenters* have used the instrument for this purpose, but the results are of very little value. The ratio can be found with some certainty, but it is almost impossible to measure the phase-angle upon an oscillogram with any accuracy, owing to the smallness of its magnitude and the relatively broad lines by which the wave-form is traced out. The method is of interest, but of little practical use, as the oscillograph cannot be considered for this purpose an instrument of precision.

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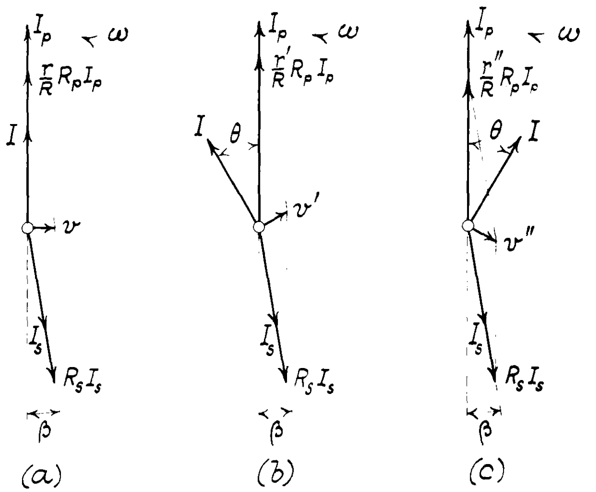
* E. Bennett, "A milliampere current transformer," Proc. Amer. I.E.E., vol. 33, pp. 625-639, 1914.



Vector diagrams for Barbagelata's null dynamometer method.

FIG. 25

CHAPTER IV.

ABSOLUTE METHODS. NULL.

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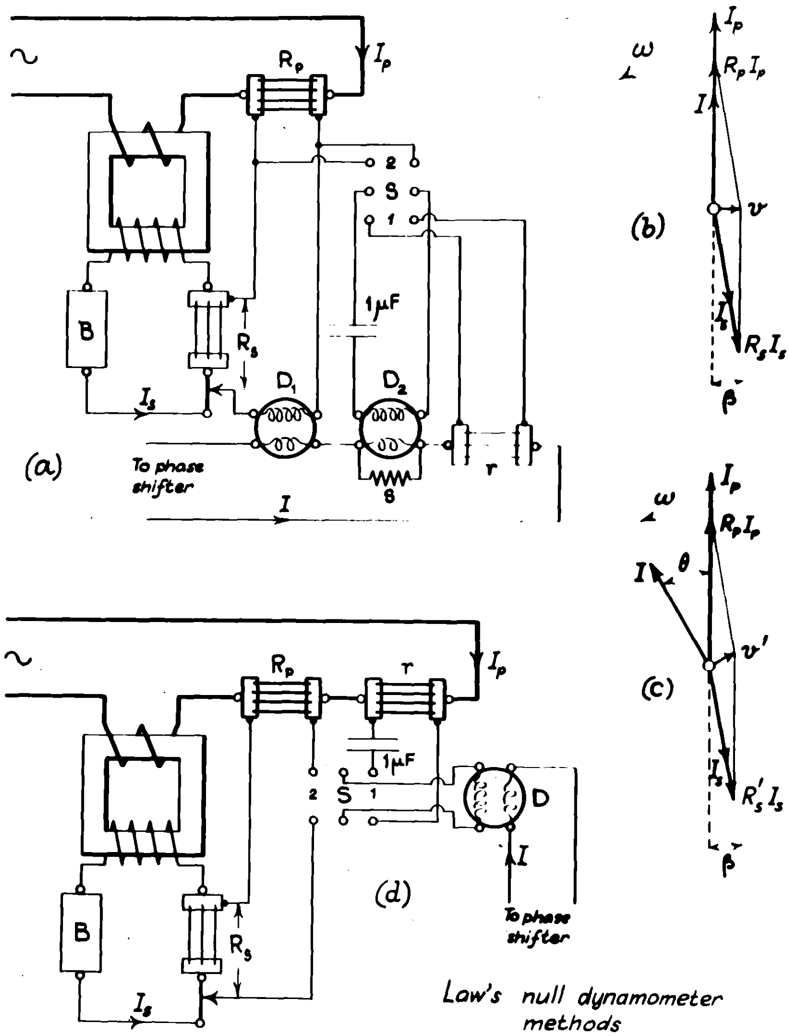
1. Methods using one or two dynamometers.

One or two methods have been devised in which the ratio and phase-angle of a transformer can be determined from the conditions required to give zero reading upon dynamometers connected in the measuring circuit. In general the methods are rather slow to use since there are usually a number of adjustments to be made; consequently they are now superseded by other more direct processes, but have, nevertheless, a considerable technical and historical interest.

Barbagelata has described a method, illustrated in Fig. 21a and described in Section 3 of the preceding Chapter, which lends itself to null operation. Referring to the circuit diagram I is first adjusted in phase with I_p and r varied until D reads zero, so that the resultant voltage v on the volt coils is in quadrature with I, as in Fig. 25a. The phase of I is then shifted through an angle θ in advance of I_p and r altered to a value r' until the resultant voltage v' is normal to I and D again reads zero as in Fig. 25b. Finally, I is adjusted to lag by θ behind I_p, r' being changed to r'' so that v'' and I are in quadrature and a null reading is again secured, as shown in Fig. 25c. From the simple geometry of these vector diagrams,



Law's null dynamometer methods

FIG. 26

$$\frac{r}{R} R_p I_p = R_s I_s \cos\beta$$
$$\frac{r'}{R} R_p I_p \cos\theta = R_s I_s \cos(\theta-\beta)$$
$$\frac{r''}{R} R_p I_p \cos\theta = R_s I_s \cos(\theta+\beta)$$

From the first, $K_c = \frac{I_p}{I_s} = \frac{R}{r}\cdot\frac{R_s}{R_p}\cos\beta \doteq \frac{R}{r}\cdot\frac{R_s}{R_p}$

Subtracting the third from the second and dividing by the first gives

$$\tan\beta = \frac{r'-r''}{2r\tan\theta} \doteq \beta$$

In particular, if $\theta = 45^\circ$, $\tan\theta = 1$ and

$$\tan\beta = \frac{r'-r''}{2r} \doteq \beta$$

Hence the ratio and phase are determined in terms of the three balance settings required to maintain D at zero reading.

A second method, resembling the deflectional method of Agnew and Fitch (see Fig.16b) is due to Laws* and is illustrated in Fig.26a. It differs from the preceding in that the balancing adjustments are made in the secondary circuit, while two dynamometers are used, one as a detector and one to enable the phase of the auxiliary supply to be adjusted to the desired position. The purpose of D_2 is to enable the current I to be set in phase with I_p, and so that the setting can be made with the greatest precision a condenser is put in the volt coils of the dynamometer to shift the phase of the current therein by 90° relative to the applied voltage. The desired adjustment can then be made at zero reading of D_2 instead of/

* F.A.Laws, "Determination of constants of instrument transformers," __Elec.World__, vol.55, pp.223-224, 1910.

of at a maximum reading, with resulting increase in precision. With the switch S in the position 1 the voltage applied to the voltage circuit of D_2 is rI and the volt coil current leads nearly 90° thereon. The slight lag of the current in the current coil relative to rI can be adjusted by the shunt s until the currents in the volt and current coils are in quadrature and D_2 reads zero. The switch S is then put in the upper position 2, so that the voltage on the volt-coil circuit of D_2 is $R_p I_p$, the phase shifter being regulated to make the deflection again zero; then I and I_p are in phase. It is now possible by regulation of R_s — which includes a low resistance slide-wire or consists of a four-terminal resistance shunted by a plug box — to reduce the deflection of D_1 to zero, the vector relations being shown in Fig. 26b, whence

$$R_p I_p = R_s I_s \cos\beta,$$

and

$$K_c = \frac{R_s}{R_p}\cos\beta \doteq \frac{R_s}{R_p}.$$

Laws does not state how to find β ; this can be readily done by advancing the phase of I through a known angle θ and readjusting R_s to a value R_s' to make the reading of D_1 again zero, as indicated by the vectors of Fig. 26c. Then,

$$R_p I_p = R_s' I_s \cos(\theta-\beta)$$

From the two relationships,

$$\tan\beta = \frac{R_s - R_s'}{R_s' \tan\theta} \doteq \beta$$

or for $\theta = 45°$, $\tan\beta = \dfrac{R_s - R_s'}{R_s'} \doteq \beta$

Since four separate adjustments are required the method is somewhat slower than the preceding; Laws has described a modification in which greater speed of working can be attained. Referring to Fig.26d the switch S is put to the right and the phase shifter adjusted to make D read zero; then I is very nearly in phase with I_p . With S to the left R_s is adjusted to restore D to zero; then as before $R_p I_p = R_s I_s \cos\beta$. The current I is then advanced by θ and R_s altered to give zero deflection, so that $R_p I_p \cos\theta = R_s' I_s \cos(\theta - \beta)$. Thus three settings are requisite, and only one dynamometer need be provided. Laws has constructed a special instrument of string galvanometer type for the purpose.

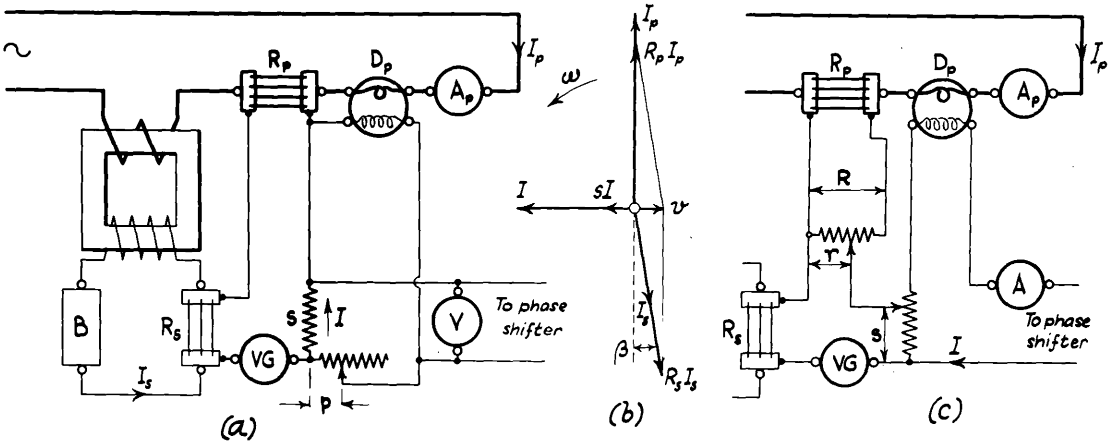
2. <u>Method using dynamometer and vibration galvanometer.</u>

A method has been described in which a dynamometer is used to enable the phase of an auxiliary current to be adjusted while the balance of the secondary current against the primary current is indicated by means of a vibration galvanometer or other type of a.c. detector. The method is easy to manipulate and is capable of determining Kc within 0.1% and β to about the nearest minute, with care.

The method of de la Gorce,* used in the Laboratoire Centrale de l'Electricité, is shown in Fig.27a, the principle being to balance the resultant of $R_p I_p$ and $R_s I_s$ in magnitude and phase, so that the voltage across the galvanometer is zero, by the voltage drop through an auxiliary resistance

carrying/

* P. de la Gorce, "Phase lag in current transformers," Elecn., vol.78, pp.463-465, 1917. Also Bull.Soc.Int.des Elecns., vol.6, pp. 299-307, 1916.



De la Gorce's null method

FIG. 27.

carrying a current I at balance. Neglecting the reactance of the volt coil of D_p, it is clear that the current therein and the current I will be in phase; if the phase shifter be adjusted to make D_p read zero then I_p and I will be in quadrature, as shown in Fig.27b. If now either R_p or R_s and s or p be adjusted until the vibration galvanometer is undeflected then ϑ, the resultant of $R_p I_p$ and $R_s I_s$ will be equal and opposite to sI so that

$$R_s I_s \cos\beta = R_p I_p$$

and $$\tan\beta = \frac{sI}{R_p I_p}$$ numerically.

If E be the reading of the voltmeter V, $I = E/(s+p)$ giving

$$K_c = \frac{R_s}{R_p}\cos\beta \doteqdot \frac{R_s}{R_p}$$

$$\tan\beta = \frac{Es}{(s+p)} \cdot \frac{1}{R_p I_p},$$

I_p being indicated by A_p.

Since it is not an easy matter to construct an adjustable four-terminal low resistance it is convenient in practice to use the arrangement shown in Fig.27c. The resistance R_p is shunted by a resistance box R with a travelling plug by which a tapping r/R can be obtained. The resistance s has a similar moving contact. Barbagelata* has shown that the effect of volt coil reactance on the exact setting of quadrature between I and I_p in Fig.27a can be completely eliminated by putting s and the volt coil in series and measuring I by means/

* Barbagelata, loc.cit.ante., 1921.

means of an ammeter A. Balance is then secured by adjustment of r and s; so that

$$R_s I_s \cos\beta = \frac{r R_p}{(R+R_p)} I_p,$$

$$\tan\beta = \frac{s I}{\frac{r R_p}{(R+R_p)} I_p},$$

from which

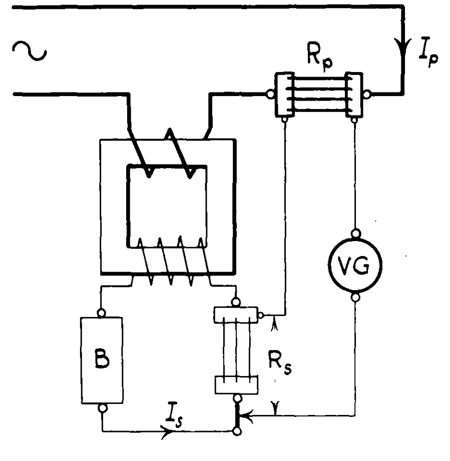
$$K_c \doteq \frac{(R+R_p)}{r} \cdot \frac{R_s}{R_p} \doteq \frac{R}{r} \cdot \frac{R_s}{R_p}$$

and

$$\tan\beta = \frac{(R+R_p)}{r} \cdot \frac{s}{R_p} \cdot \frac{I}{I_p} \doteq \frac{R}{r} \cdot \frac{s}{R_p} \cdot \frac{I}{I_p} \doteq \beta$$

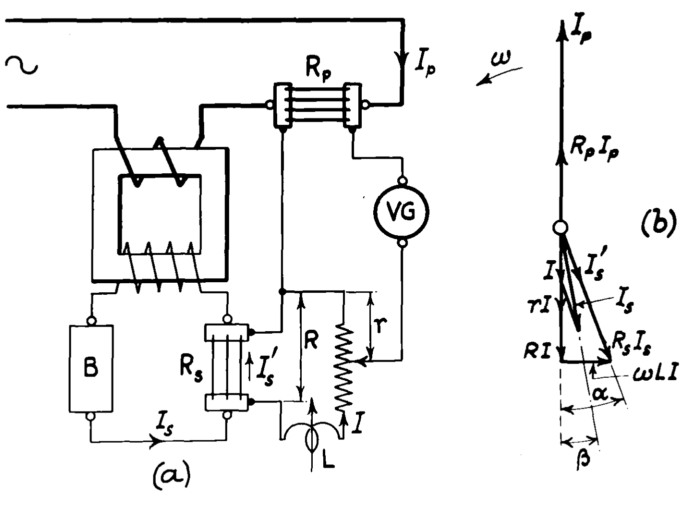
3. Methods without dynamometers. Resistances in both circuits.

In the preceding methods the balancing of the primary magnitude against the secondary magnitude has been effected by resistance adjustments while the opposition of phase has been regulated with the aid of a phase shifter and dynamometer. Complete balance has been indicated in the methods of Section 1 by a dynamometer and in those of Section 2 by a vibration galvanometer. An important series of methods is now to be considered in which the adjustment of magnitudes is again made by changes in resistances but in which the phase relation between the magnitudes is compensated without the use of phase shifter and dynamometer. The methods partake of some of the characteristics of an alternating current bridge since balance is secured by means of a suitable network of properly adjusted impedances and is usually indicated by a vibration galvanometer or other a.c. detector.



∼
I_p
R_p
VG
B
R_s
I_s

FIG. 28.



∼
I_p
R_p
VG
B
R_s
I_s'
I_s
R
r
I
L
(a)
ω
I_p
R_pI_p
(b)
I
I_s'
rI
I_s
RI
R_sI_s
ωLI
α
β

FIG. 29

3a. Phase angle uncompensated. - The principle of all methods of this class is easily understood by consideration of the circuit shown in Fig.28. A four-terminal resistance R_p is put into the primary circuit and a second similar resistance in the secondary; R_s must be adjustable, either by shunting or by consisting in part of a low resistance slide wire. If I_p and I_s were exactly in opposition of phase it would clearly be possible to reduce the deflection of the galvanometer to zero by adjustment of R_s until $R_p I_p = R_s I_s$.

Since, however, I_s leads on I_p reversed by a small angle β it is only possible to get a minimum deflection of the detector by adjustment of R_s alone. In order to secure a true balance it is essential to introduce by means of some reactive device an e.m.f. in quadrature with one or other of the resistance drops so that the phase-angle β can be allowed for and a null reading obtained on the detector. This can be done in a variety of ways, some of greater practical interest than others, which it is the object of the following sub-sections to consider.

3b. Phase angle compensated by L. - The required reactive component can be most obviously provided* by inserting a self inductance in the circuit, as shown in Fig.29a. Balance is attained by adjusting r and L successively until the V.G. remains undeflected. The resistance R of the compensating mesh conveniently consists of a slide wire of about 100 ohms and/

* C.H.Sharp, Trans.Amer.I.E.E., vol.28, pp.1040-1052, 1910.

and includes in computations the small resistance of the variable self inductance L. Assuming balance to have been attained it is clear that the voltage $R_p I_p$ is equal and opposite to the value of rI; hence I is in opposition to I_p as shown in Fig. 29b. The secondary current I_s is compounded of I_s', the current in R_s, and I, the current in the compensating circuit. The voltage drop $R_s I_s'$ over R_s is equal to that over the compensating circuit, the components being RI and $\omega L I$ therein. From the simple geometry of the diagram resolving the current triangle gives

$$I_s \cos\beta = I + I_s' \cos\alpha,$$
$$I_s \sin\beta = I_s' \sin\alpha,$$

and the condition of balance makes

$$R_p I_p = rI.$$

But $\cos\alpha = RI/R_s I_s'$ and $\sin\alpha = \omega L I/R_s I_s'$ so that

$$I_s \cos\beta = I\left(1+\frac{R}{R_s}\right),$$

$$I_s \sin\beta = \frac{\omega L}{R_s} I;$$

whence, $$\tan\beta = \frac{\omega L}{R+R_s} \doteq \beta$$

Again, using the balance condition makes

$$K_c = \frac{R_s}{R_p}\cdot\frac{r}{R+R_s}\cos\beta \doteq \frac{R_s}{R_p}\cdot\frac{r}{R+R_s}.$$

It/

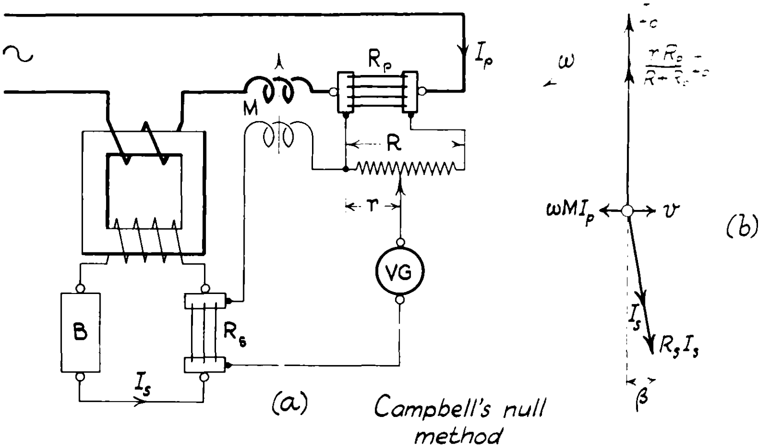

I_p
R_p
ω
M
R
r
VG
B
R_6
I_s
(a)
$\omega M I_p$
v
(b)
I_s
$R_s I_s$
β
Campbell's null method

FIG. 30

It should be noted that for balance $R_p I_p$ is less than $R_s I_s$; hence the secondary resistance R_s must be chosen to give a drop in excess of that over the primary resistance before balance is possible. There may thus be some danger of an excessive addition to the secondary burden. The principal defect of the method, however, lies in the fact that the apparatus is peculiarly susceptible to error due to stray fields influencing the compensating circuit; this can be minimised by careful arrangement of the circuit and by the use of an astatic inductance for L . The method is only capable of measuring positive or leading values of β.

3c. <u>Phase angle compensated by M between primary and detector circuits.</u>

Mutual inductances have the advantage in practice of being capable of regulation down to zero and if necessary of being reversed in sign; they are therefore usually preferred to self inductances in most testing work. Campbell* has described a method in which the desired quadrature e.m.f. is supplied by a variable mutual inductance M with one of its windings in the primary circuit of the transformer, as shown in Fig.30a. The resistance R_p is a standard four-terminal resistance with a slide-wire shunt, and balance is secured by adjustment of r and M . From the vector diagram of Fig.30b it is clear that v is opposed by $\omega M I_p$ and that

$$R_s I_s \cos\beta = \frac{r R_p}{R+R_p} I_p,$$

$$\frac{r R_p}{R+R_p} I_p \tan\beta = \omega M I_p.$$

* A.Campbell, "On the use of mutual inductometers," Proc.Phys.Soc. 1910. See also A.Barbagelata, loc.cit.ante, 1921.

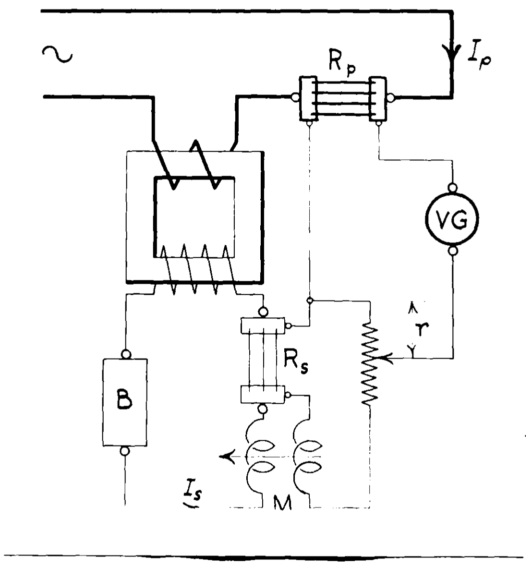


FIG. 31.

From these equations

$$K_c = \frac{R+R_p}{r} \cdot \frac{R_s}{R_p} \cos\beta \doteq \frac{R+R_p}{r} \cdot \frac{R_s}{R_p} \doteq \frac{R}{r} \cdot \frac{R_s}{R_p},$$

$$\tan\beta = \frac{\omega M (R+R_p)}{r R_p} \doteq \frac{\omega M R}{r R_p} \doteq \beta,$$

since R_p is small in comparison with R. Both positive and negative values of β can be measured since M can have either sign.

The great practical disadvantage of this method is that the primary of the variable mutual inductance must be capable of carrying the full primary current of the transformer; it is not easy to construct a mutual to fulfil this condition.

3d. Phase angle compensated by M between secondary and detector circuits.

The disadvantage of the preceding method can be overcome by inserting the mutual inductance in the secondary circuit; its primary winding then never carries more than 5 amperes and a compact piece of apparatus can easily be constructed. Two methods using this principle have been proposed and will now be described.

Sharp* has suggested the method shown in Fig.31 which is the mutual inductance analogue of Fig.29a. Balance is obtained by adjustment of r and M. Little service is done, however, by working out the somewhat complex balance conditions since the application of mutual inductance in the secondary circuit can be made in a much simpler way, suggested by the same investigator.

In 1910 Sharp described the method shown in Fig.32a.

In/

* C.H.Sharp, loc.cit., 1910.

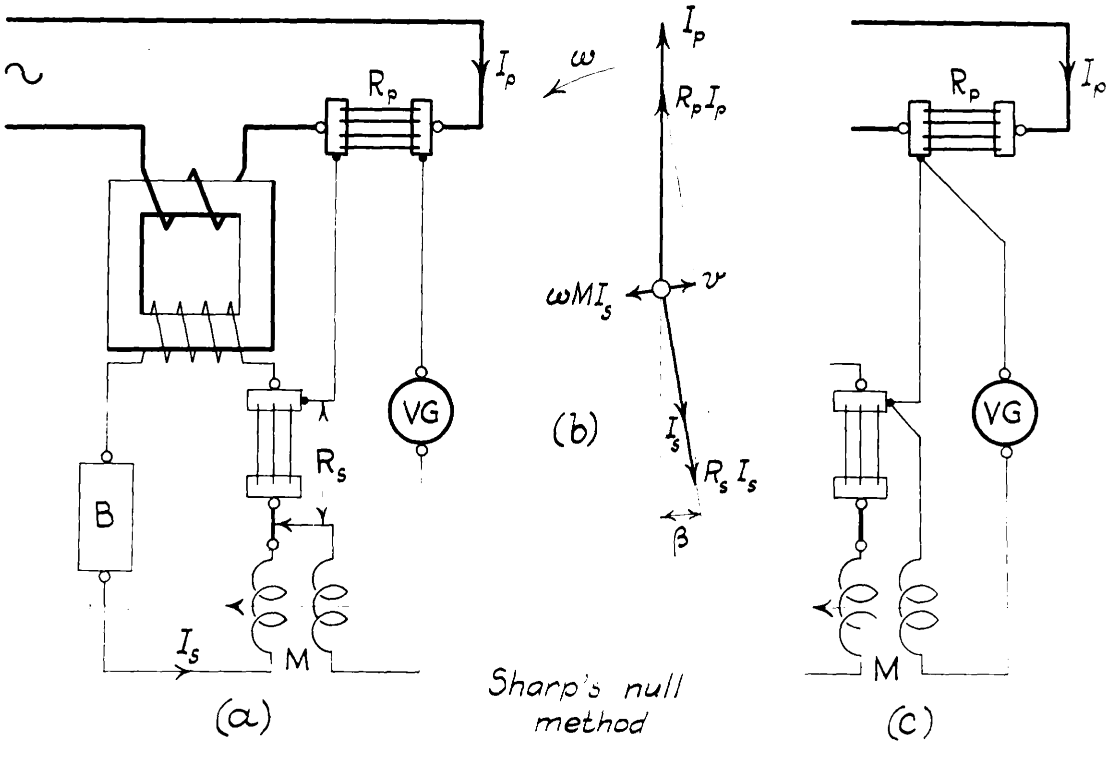

~
I_p
R_p
ω
$R_p I_p$
$\omega M I_s$
v
VG
(b)
R_s
I_s
$R_s I_s$
β
B
M
I_s
(a)
Sharp's null
method
(c)

FIG. 32.

In this R_p and R_s are low resistances of approximately equal volt-drop - about 0.25 to 1 volt - R_s consisting of a fixed four-terminal resistance in series with a low resistance slide wire, or of a low resistance shunted by a plug box. Balance is secured by varying R_s and M until the galvanometer is undeflected; then, as the vector diagram of Fig.32b shows, the resultant volt drop v across the shunts is normal to I_s and is equal and opposite to $\omega M I_s$. Then

$$R_p I_p \cos\beta = R_s I_s ,$$
$$R_p I_p \sin\beta = \omega M I_s ,$$

whence $$K_c = \frac{R_s}{R_p \cos\beta} \doteqdot \frac{R_s}{R_p} ,$$

and $$\tan\beta = \frac{\omega M}{R_s} \doteqdot \beta .$$

The simplicity and speed of the method make it one of the most important so far devised. Sharp and Crawford* in 1911 described tests made by use of the method and gave full details of their apparatus. These investigators claimed a precision of 1 part in 25000, using a drop of 0.125 volt over R_s at 5 amperes; the detector was a d.c. galvanometer working in conjunction with a synchronous rectifying key. The resistance R_s and the inductor M formed a self-contained unit; R_s consisted of a non-reactive resistance of manganin strip/

* C.H.Sharp and W.W.Crawford, "Some recent developments in exact alternating current measurements," Trans.Amer.I.E.E. vol.29, pp.1517-1541, 1911.

strip together with a slide wire; M was of the disc type and was astatically wound. The method has been adopted at the Bureau of Standards, the procedure in that laboratory being described in 1912 by Agnew and Silsbee,* who used a vibration galvanometer as the balance detector.

Apart from the advantages of simplicity and speed the method has a number of practical advantages. It is flexible and of almost unlimited range, so that it is very suitable for general accurate work in the test-room. The secondary burden introduced by R_s and the primary of M is slight, being of the order of 0.1 ohm or 1 to 2 volt amperes. Since all transformers have now the standard 5 ampere secondary winding a single secondary resistance and mutual inductor serve for all tests. Drysdale points out that R_s may conveniently be a resistance of about 0.04 ohm shunted by a plug resistance box; with this value of R_s , an inductor giving about 5 microhenrys for M suffices to measure phase-angles up to 2° at 50 cycles per second. A suitable series of primary resistances must be provided to meet the various values of nominal ratio found in practice.

The method is subject to certain sources of error, to which attention must now be drawn. The resistances R_p and R_s must be designed so as to have negligible residual inductances, or alternatively their residuals must either be/

* P.G.Agnew and F.B.Silsbee, "The testing of instrument transformers," Trans.Amer.I.E.E., vol.31, pp.1635-1638, 1912. See also R.S.J.Spilsbury, loc.cit.ante., 1920; F.B.Silsbee, loc.cit.ante, 1924; C.V.Drysdale, "The testing of current transformers," Journal Sci. Insts., vol.3, pp.57-58, 1925, for various details.

be equal or known.* Inductive influence, particularly between the primary circuit and the inductor M may also cause error. This can be avoided by careful arrangement of connections and by the use of a properly designed astatic inductor.† The presence of inductive interference can be easily detected ‡ by rearranging the connections as shown in Fig.32c, setting M at zero, and then passing full primary circuit. If there is any stray inductance error the galvanometer will deflect; this deflection can then be reduced to zero by judicious alterations in the relative positions of the various portions of the test circuit, after which the original connections may be resumed and the test on the transformer carried out. A further possible source of error is due to capacity and leakage currents flowing from the primary circuit and into the detector <u>via</u> the interwinding capacity and insulation resistance of M; careful attention to the potential to which the set-up is subjected enables this error ~~to be made~~ negligible.

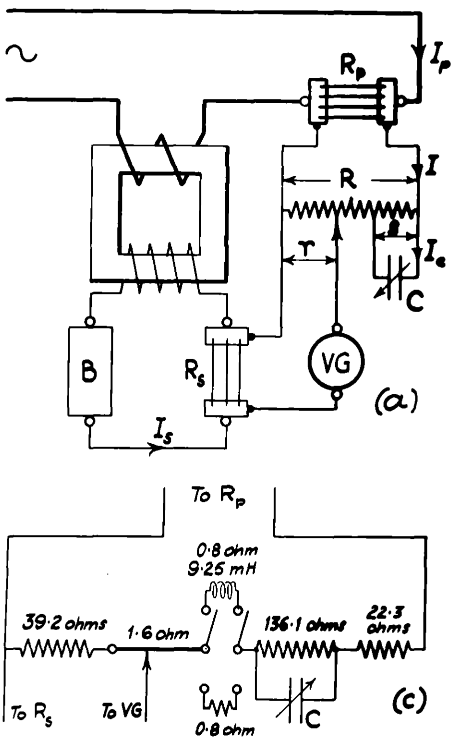
3e. <u>Phase angle compensated ~~by~~ C.</u> The trouble due to inductive interference in the preceding method can be completely avoided by compensating the phase angle by use of a condenser instead of by a mutual inductance. Schering and Alberti** have described/

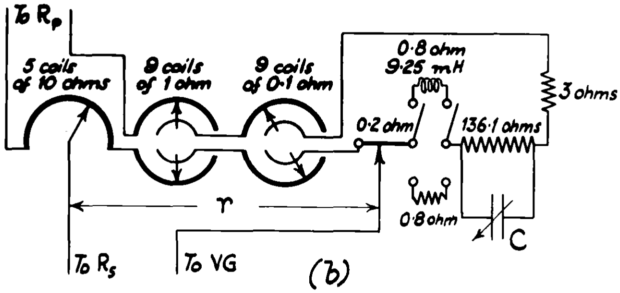
* The theory of the method including residual effects is given in F.A.Laws, <u>Electrical Measurements</u>, Chapter XII, pp.581-583, 1917.

† H.B.Brooks and F.C.Weaver, "A variable self and mutual inductor," <u>Bull.Bur.Stds.</u>, vol.13,pp.569-580, 1917.

‡ J.M.Stein, <u>Trans.Amer.I.E.E.</u>, vol.43,pp.294-297, 1924.

** H.Schering & E.Alberti,"Eine einfache Methode zur Prüfung von Stromwandlern," <u>Arch.f.Elekt.</u>, vol.2,pp.263-275, 1914. Also A.Barbagelata, <u>loc.cit.ante</u>, 1921.





R_p ohms	R_s ohms						
	0·02	0·024	0·03	0·04	0·05	0·06	0·08
1·0				1/5			2/5
0·2	2·5/5	3/5		5/5		7·5/5	10/5
0·05			15/5	20/5	25/5	30/5	40/5
0·01	50/5	60/5	75/5	100/5	120/5	150/5	200/5
0·002	250/5	300/5		500/5	600/5	750/5	1000/5
0·0005		1200/5	1500/5	2000/5			

Schering and Alberti's null method

FIG. 33

described an excellent method, shown in Fig.33a, which has been adopted for routine testing of transformers at the Reichsanstalt and which excels all others in simplicity and ease of operation. Referring to the diagram, $R_p I_p$ is chosen 4 to 8 times greater than $R_s I_s$ and the resistance R_p is shunted by R, of about 200 ohms. The phase of the current I is adjusted by means of a condenser C shunting a portion s of R and the fraction r/R is also altered until the volt drop rI is equal and opposite to $R_s I_s$. Balance is thus secured by varying r and C. The theory of the method, in its complete form, is far from easy but certain approximations are admissible which greatly simplify the rather complex general expressions. Proceeding with the rigorous solution, so far as it is required, let i_p, i_s, i, and i_c be the harmonic vectors of the various currents in the network at balance. Then by application of Kirchloff's rule,

$$r i = R_s i_s$$

$$(R+R_p) i - R_p i_p - s i_c = 0$$

$$\left(s + \frac{1}{j\omega C}\right) i_c - s i = 0$$

in the three meshes of the detector circuit. Eliminating i and i_c

$$i_p = \left\{ (R+R_p)\frac{R_s}{R_p r} - \frac{R_s}{R_p}\cdot\frac{s}{r}\cdot\frac{\omega^2 C^2 s^2}{(1+\omega^2 C^2 s^2)} - j\,\frac{R_s}{R_p r}\cdot\frac{\omega C s^2}{(1+\omega^2 C^2 s^2)} \right\} i_s,$$

where/

where ω is the pulsatance of the currents and j the operator rotating a vector through $+\pi/2$. From this equation, which may be written, $i_p = (a - jb)i_s$
the ratio $I_p/I_s = (a^2+b^2)^{1/2}$ and the angle between I_s and I_p reversed is arctan (b/a). The resulting expressions are rather unwieldy. However in the apparatus designed by Schering and Alberti $\omega^2 C^2 s^2$ can be neglected in comparison with unity; moreover, in the worst case possible the neglect of the second term only introduces an error of 8 in 10000 in the resistance component of the above impedance operator and is usually much less. Neglecting the second term in comparison with the first makes

$$i_p \doteqdot \left[(R+R_p)\frac{R_s}{R_p r} - j\frac{R_s}{R_p r}\cdot\omega C s^2\right] i_s$$

very nearly. Again $\omega C s^2$ is small in comparison with $(R+R_p)$ - usually in the proportion of about 6 or less to 2000 - and has little effect on the magnitude of the impedance operator. Hence

$$K_c = \frac{I_p}{I_s} \doteqdot \frac{R_s}{R_p r}(R+R_p),$$

$$\tan\beta \doteqdot \frac{\omega C s^2}{(R+R_p)} \doteqdot \beta$$

In most cases R_p is small in comparison with R so that finally

$$K_c \doteqdot \frac{R_s}{R_p}\cdot\frac{R}{r},$$

$$\beta \doteqdot \frac{\omega C s^2}{R}.$$

Schering and Alberti describe a self-contained detector circuit shown in Fig.33b. The resistance R is made up of three dials of resistance coils the first containing 5 coils of 10 ohms, the second 9 coils of 1 ohm, and the third 9 coils of 0.1 ohms; the last two are compensated on the well-known Feussner principle. In series with these are a 0.2 ohm slide wire, a resistance of 136.1 ohms, a resistance of 0.8 ohms, and a final resistance of 3 ohms, making the total value of R equal to 200 ohms. The resistances are contained in a suitable box provided with the necessary dial switches and terminals. The condenser C is a mica standard of the decade pattern adjustable to 1 microfarad by steps of 0.001 μF. When shunted across the 136.1 ohm resistance 1 μF enables an angle of 100 minutes to be compensated at 50 cycles per second. When testing a transformer on heavy inductive burden β may become negative; such angles are measured by substituting for the 0.8 ohm resistance a small inductance of equal resistance. This inductance consists of a separate astatically wound coil having an inductance of 9.25 millihenrys which balances an angle of -50 minutes at 50 cycles. Resistance or inductance can be inserted at will by means of a throw-over switch. The leads connecting the compensation apparatus to R_s and C are of little importance; those to R_p must be short and thick, and should not exceed 0.04 ohm if error is to be avoided. The standard resistances R_p and R_s are of doubled manganin strip for lower currents and of tubular water-cooled construction for higher currents; they have been described in Chapter III of Part I.

The procedure for attaining balance is very simple. With the Vibration galvanometer shunted and $C = 0$ adjust r until a minimum deflection is obtained; if this cannot be secured by the rough adjustment of r the voltages to be balanced are not in opposition. Reverse the connections to R_p and repeat the adjustment of r, gradually increasing the sensitivity. Complete the balance of alterations of the position of the contact on the slide wire and of the condenser. If balance cannot be secured by use of the condenser insert the inductance coil and proceed as before. The angle is then calculated from

$$\tan\beta \doteq \beta \doteq \left(\frac{\omega C s^2}{R} - \frac{\omega L}{R} \right)$$

It/

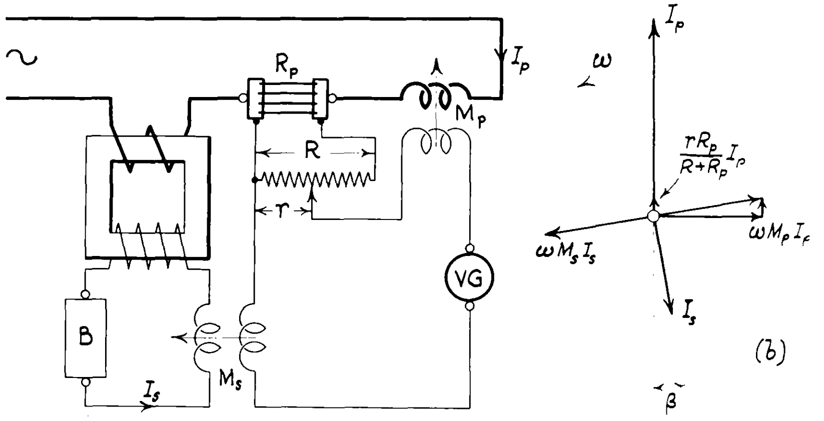
It is possible to carry out tests with a much simplified apparatus. From above

$$K_c \doteqdot \frac{R_s}{R_p} \cdot \frac{R}{r} = K_{nc}\left[\frac{R_s}{R_p} \cdot \frac{R}{K_{nc}}\right]\frac{1}{r} = K_{nc}\,\frac{K}{r}$$

where K_{nc} is the nominal ratio of the transformer. For each value of the nominal ratio and each chosen R_p it is possible to choose R_s so that with R = 200 ohms, K has a constant value of say 40. Then $R_s = 0.2\,K_{nc}\,R_p$. Moreover, as the actual ratio rarely differs more than $\pm 2\%$ from the nominal value and K is 40, r need only be variable between 39.2 and 40.8 ohms. The above described compensating apparatus may then be replaced by the simpler circuit of Fig. 33c. Appropriate primary and secondary resistances are detailed in the table in Fig.33 for various nominal ratios occuring in practice.

4. Methods without dynamometers. Resistance in one circuit.

It is now necessary to examine certain methods in which a single resistance is used, connected alternatively in the primary or the secondary circuit, while electromotive forces proportional to the primary and secondary currents are injected into the detector circuit by mutual inductances linking that circuit with those of the transformer under test. These methods have the advantage, not possessed by any previous method, that there is no direct electrical connection between the primary and secondary circuits; consequently, error due to capacity and leakage currents flowing from the primary supply/



Barbagelata's mutual inductance null method

FIG. 34.

supply to the detector are quite negligible.

4a. <u>Resistance in the primary circuit</u> - The method shown in Fig.34a has been described by Barbagelata[*]. Two mutual inductances M_p and M_s link the primary and secondary circuits with the detector; one winding of M_p must be suitable for the primary current I_p and is not very easy to construct. One winding of M_s carries I_s , up to 5 amperes; both mutuals are preferably variable though it is only essential that one, M_p say, should be so. The required in-phase component of voltage is obtained from a shunted four-terminal resistance, as shown. Balance is obtained by varying r and M_p , with or without adjustment of M_s ; the vector relations shown in Fig.34b hold at null indication. Then

$$\omega M_s I_s \sin\beta = \frac{r R_p}{R+R_p} I_p ,$$

$$\omega M_s I_s \cos\beta = \omega M_p I_p ,$$

whence

$$K_c = \frac{M_s}{M_p}\cos\beta \doteq \frac{M_s}{M_p} ,$$

$$\tan\beta = \frac{r R_p}{\omega M_p (R+R_p)} \doteq \frac{r}{R}\cdot\frac{R_p}{\omega M_p} \doteq \beta .$$

Beyond the difficulty of making M_p the method is both simple and quick, and makes use of ordinary apparatus. Although the capacity and leakage effects are negligible, inductive interference is considerable unless the inductors are made quite astatic.

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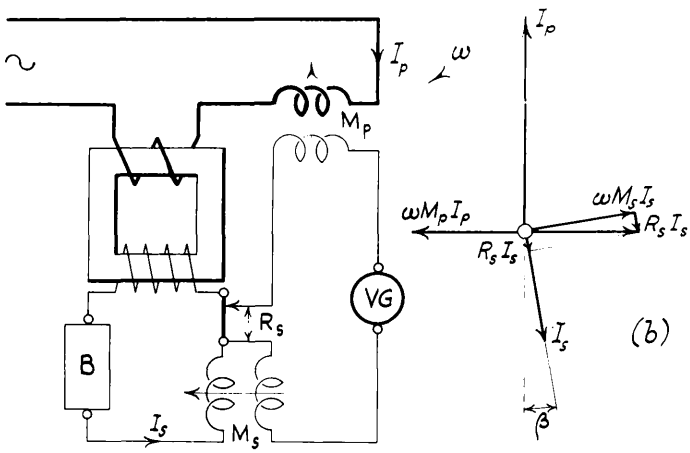
[*] A.Barbagelata, <u>loc.cit.ante.</u>, 1921.



Fortescue's mutual inductance null method

FIG. 35.

4b. <u>Resistance in the secondary circuit</u>. - A second method of connection, suggested by Sharp and Crawford,* is shown in Fig.35a. If this diagram be compared with Fig.32a it will be seen that the mutual inductances replace the two resistances therein used to compare the primary and secondary currents, while the phase-angle is now compensated by a resistance instead of by a mutual inductance. A similar reciprocal relation between mutual inductance and resistance exists between the preceding method Fig.34a and the method of Fig.30a.

By adjusting M_p or M_s and R_s balance will be secured when the vector relations of Fig.35b are satisfied; that is,

$$\omega M_p I_p \cos\beta = \omega M_s I_s,$$
$$\omega M_p I_p \sin\beta = R_s I_s,$$

whence

$$K_c = \frac{M_s}{M_p \cos\beta} \doteq \frac{M_s}{M_p},$$
$$\tan\beta = \frac{R_s}{\omega M_s} \doteq \beta.$$

The advantages of the method are many. Since it is much easier to construct perfect mutual inductances - <u>i.e.</u>, in which the secondary e.m.f. is in quadrature with the primary current - then to make non-reactive low resistances, any error which would arise in measuring β in consequence of residuals is removed by substituting mutuals for resistances. The/

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* C.H.Sharp and W.W.Crawford, loc.cit., 1911; R.S.J.Spilsbury, <u>loc.cit.</u>, 1920, F.B.Silsbee, <u>loc.cit</u>., 1924.

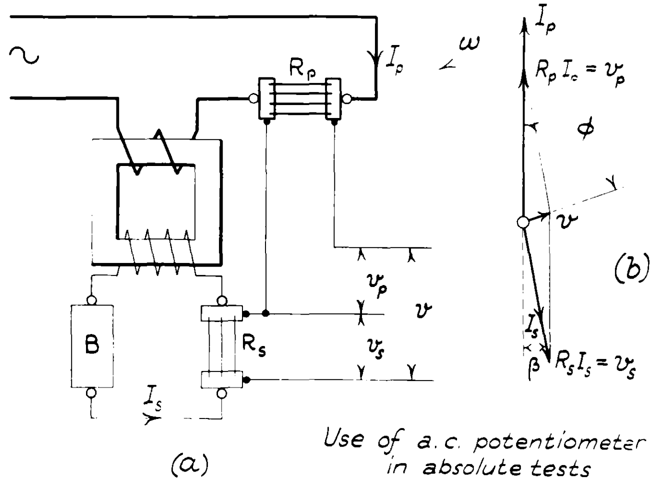
The secondary burden is small, being of the order of 4 volt-amperes, while at the same time a large e.m.f. may be impressed on the galvanometer circuit with resulting high sensitiveness; the voltage at the mutual secondaries is of the order of 4 volts. Again, since there is no connection between the primary and secondary circuits capacity and leakage effects in the detector circuit are absent.

The principal defect of the method is that the inductors must be astatic so that they can neither influence one another nor be affected by stray fields from the transformer under test. Fortescue* has overcome this objection by constructing the inductors in toroidal ring-wound form. The primary and secondary coils of each inductor are wound upon marble rings of circular cross-section, accurately machined and wound with uniformly distributed coils. The inductor M_s consists of three toroidal mutuals connected in series, each having tapped windings so that the inductance can be regulated in fine steps from a maximum value of 3.0222 millihenrys; M_p consists of a single toroid of 0.8265 millihenrys maximum mutual inductance. Full particulars of the complete set-up are given in his paper, with a detailed discussion of its uses in practice. The method is very flexible and a wide range of primary currents up to 5000 amperes is covered.

5. A.C.Potentiometer method.

The apparatus described in the preceding Sections 3 and 4 constitutes/

* C.L.Fortescue, "The calibration of current transformers by means of mutual inductance," Proc.Amer.I.E.E., vol.4, pp.1199-1215, 1915.



Use of a.c. potentiometer in absolute tests

FIG. 36

constitutes, in a variety of different ways, a simple type of alternating current potentiometer of limited range by means of which two voltages, proportional respectively to the primary and secondary currents of the transformer, may be compared and the phase displacement between them determined. There are, however, a number of types of a.c. potentiometer intended for use in a wide range of a.c. measurements, the best known being that designed by Dr C.V.Drysdale[*] and constructed by H.Tinsley. A description of the instrument is here superfluous but it will suffice to state that by its use alternating voltages up to about 1.5 volts may be measured and the phase-angles between such voltages obtained, in each case by direct readings on the dials of the potentiometer.

Such an instrument is expensive and not frequently available; however, where an a.c. potentiometer is obtainable it forms, in conjunction with a set of non-reactive, four-terminal resistances, a ready and precise means of measuring current transformer ratio and phase.[†] Referring to Fig.36a, a four-terminal resistance is put into each circuit of the transformer; for a ratio of 50/5 these may be of 0.01 ohm and 0.1 ohm respectively in primary and secondary, and in any other case should similarly be chosen to give equal

volt/

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* C.V.Drysdale, "The use of the potentiometer on alternate current circuits," <u>Proc.Phys.Soc.</u>, vol.21, pp.561-572, 1910.

† <u>C.L.Dawes</u>, <u>Proc.Amer.I.E.E.</u>, vol.34, pp.927-940, 1915; D.C. Gall, "Testing transformers by the alternating current potentiometer," <u>Elecn.</u>, vol.83, pp.603-604, 1920; A.C.Jolley, "Some tests on modern current transformers," <u>Journal Sci. Insts.</u>, vol.3, pp.43-50, 1925; C.V.Drysdale, "The testing of current transformers," <u>idem</u>, pp.57-58, 1925.

volt-drops of the order of 0.5 volt. The resistances must be non-reactive, or alternatively of known or equal time-constants. Referring to Fig.36b measurements can be made of $v_p = R_p I_p$, $v_s = R_s I_s$, and β directly on the potentiometer. Since β is a small angle, and therefore not readable with great precision without the aid of some magnification, it is best to measure v and ϕ in its stead. Then

$v_s \sin\beta = v \sin\phi$ and

$$K_c = \frac{v_p}{v_s} \cdot \frac{R_s}{R_p}$$

$$\sin\beta = \frac{v}{v_s} \sin\phi \doteq \beta$$

The method is capable of high accuracy and is quick to use. The secondary burden is small, of the order of 2 to 3 volt-amperes. The principal disadvantage of the method is that it requires the use of an expensive and not readily obtainable instrument.

6. <u>Thermal detector methods.</u>

A certain limited use has been made of thermal methods of comparing the primary and secondary currents of a transformer, and were it not that there are so many excellent methods already available it would seem quite worth while to develop the use of thermal detectors owing to their complete immunity from troubles inherent in inductive apparatus. The methods that have been suggested differ in no essential particular from those described, except in so far as a thermal detector/

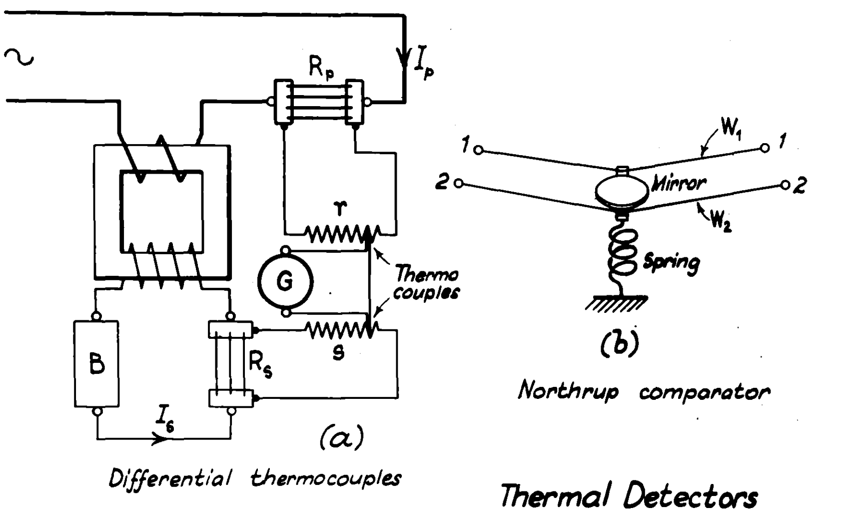
~
R_p
I_p
r
G
Thermo couples
B
R_s
s
I_s
(a)
Differential thermocouples
1
1
W_1
2
2
Mirror
W_2
Spring
(b)
Northrup comparator
Thermal Detectors

FIG. 37.

detector replaces the dynamometer or vibration galvanometer. Moreover, the published results deal with the measurement of ratio only, though it is possible also to measure phase-angles.

6a. Differential thermocouple. - In this method* two similar thermocouples acting in opposition through a d.c. galvanometer are heated by currents proportional to the primary and secondary currents of the transformer, as Fig. 37a indicates. By adjusting r or s until the galvanometer remains undeflected the e.m.f.s of the couples can be made equal; the temperatures of the couples are equal and therefore the rates at which heat is developed in the resistances r and s must be the same, i.e.,

$$r\left(\frac{R_p}{r+R_p}\right)^2 I_p^2 = s\left(\frac{R_s}{s+R_s}\right)^2 I_s^2,$$

$$K_c = \frac{R_s}{R_p}\cdot\frac{r+R_p}{s+R_s}\cdot\sqrt{\frac{s}{r}}.$$

B.G.Churcher has greatly developed the method and has devised and patented a means of determining β as well as K_c; his work, done in the Research Department of the Metropolitan Vickers Co., has not yet been published.

6b. Northrup comparator. - This instrument, which is really a differential hot-wire voltmeter, is shown diagrammatically in Fig.37b and was suggested by Robinson as an accurate means of measuring transformer ratio. It consists of two wires/

* L.T.Robinson, Trans.Amer.I.E.E., vol.28, p.1005, 1910.

wires W_1, W_2 attached respectively to fixed terminals 11, 22; at their middle an insulating bridge carrying a mirror is fixed. A spring pulls below the bridge and keeps the wires tight. The wire W_1 is connected in parallel with R_p and W_2 with R_s ; then if the volt drops in these are unequal the wires will carry different currents and will therefore heat unequally and the mirror will tilt. By adjusting R_p or R_s until there is no tilt the heating of the two wires will be the same. The two shunts are then switched into auxiliary d.c. circuits in which the currents are adjusted until the comparator is again undeflected. Then the ratio of the two direct currents is the same as the ratio of the two alternating; the d.c. ratio can be found by measuring the volt drops over the two resistances by means of an ordinary potentiometer. Agnew* found it possible to obtain agreement to 1 part in 8000 between the ratio found by this method and that determined by a dynamometer method.

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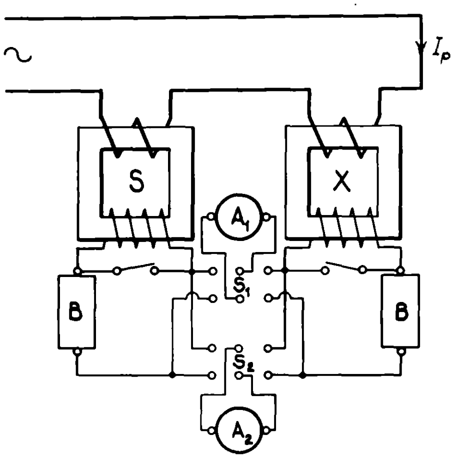
* P.G.Agnew, "A study of the current transformer with particular reference to iron loss," Bull.Bur.Stds., vol.7, pp. 423-474, 1911.



Two ammeter relative method

FIG. 38.

CHAPTER V.

RELATIVE METHODS. DEFLECTIONAL.

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1. Two ammeter method.

The ratios of two transformers of the same nominal ratio can be easily compared* by the use of two ammeters, as shown in Fig.38. S is the standard transformer for which the ratio-secondary current characteristic is known; X is the unknown transformer, its nominal ratio being equal to that of S. A_1 and A_2 are two similar ammeters, the calibrations of which need not be accurately known; they can be inserted at will into the secondary circuit of either transformer by operation of the switches S_1 and S_2. With S_1 to the left and S_2 to the right, thereby inserting A_1 in the secondary of S and A_2 in that of X, let the secondary currents of S and X be I_s and I_s' respectively. Let the readings of A_1 and A_2 be I_1 and I_2'; then if k_1 and k_2 be the correction factors for these points on the ammeter scales,

$$I_s = k_1 I_1 \quad \text{and} \quad I_s' = k_2 I_2'$$

By throwing S_1 to the right and S_2 to the left the ammeters are interchanged with respect to the transformers; if the ammeters/

* F.B.Silsbee, loc.cit., 1924.

ammeters are similar the secondary currents will be practically unaltered. The readings of the instruments will be I_2 and I_1' , so that very nearly

$$I_s = k_2 I_2 \quad \text{and} \quad I_s' = k_1 I_1'$$

From these two sets of observations,

$$I_s^2 = k_1 k_2 I_1 I_2 \qquad \text{and} \quad I_s'^2 = k_1 k_2 I_1' I_2'$$

The ratio of S at current I_s is

$$K_c = I_p / I_s$$

while that of X is

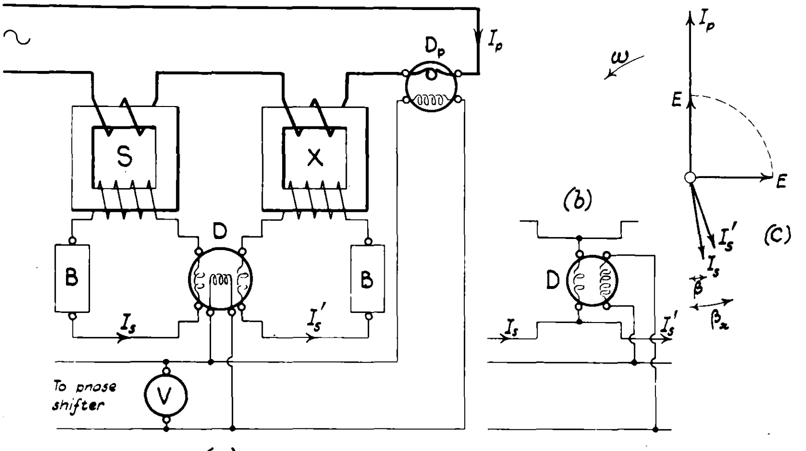
$$K_{cx} = I_p / I_s' = I_s K_c / I_s'$$

Substituting for I_s and I_s',

$$K_{cx} = \sqrt{\frac{I_1 I_2}{I_1' I_2'}} \, . K_c \ ,$$

eliminating the calibrations of the two instruments. Care must be taken in interchanging the ammeters that the secondary circuits are not opened; this can be guarded against by short-circuiting the secondaries, before operating the switches S_1 and S_2, by means of the switches shown.

The method only gives the ratio of the transformer and not the phase-angle. Moreover the accuracy falls off considerably at low currents. It is an easy and quick method for testing of ratio on site but cannot be classed as a good method for use in laboratory work.



Barbagelata's two dynamometer relative method

FIG. 39.

2. Two dynamometer Methods.

There are a number of ways of comparing the characteristics of two similar transformers by the use of two dynamometers. One such was described by Makower and Wust* and consists in connecting two transformers of the same nominal ratio back to back; the combination is then tested as a single unit by the process outlined in Section 2 of Chapter III. It is then assumed that the effective ratio of the combination is equal to the square of the ratio of the transformers and that the phase-angle of each is half the measured phase-angle of the combination; i.e., both transformers are assumed exactly similar, which is by no means warrantable in general.

An effective method† is that shown in Fig.39a. The common primary current flows in the current coil of the dynamometer D_p. The secondary currents I_s of the standard and I_s' of the unknown flow in opposition through separate current coils of a dynamometer D, or alternatively are superposed in a single current coil as in Fig.39b. The voltage coils of D_p and D are excited at voltage E from a phase shifter; they should be of equal or very low time-constant, or alternatively they may be supplied in series,

the/

* A.J.Makower and A.Wust, Elecn., vol.59, pp.581-582, 1917.

† A.Barbagelata, loc.cit.ante., 1921. See also E.C.Westcott, "Differentially wound watthour meter for testing current transformers," Elec.World, vol.76, p.433, 1920; F.B. Silsbee, loc.cit.ante., 1924.

the current in them being measured instead of the voltage. The phase shifter is adjusted until E is in phase with I_p, indicated by D_p reading a maximum, and the reading W_1 watts of D observed. E is then adjusted in quadrature with I_p, D_p then reading zero, and D is again read; if the indication be W_2 watts, Fig. 39c shows that

$$W_1 = EI_s\cos(\pi-\beta) - EI_s'\cos(\pi-\beta_x),$$
$$W_2 = EI_s\cos(\tfrac{\pi}{2}-\beta) - EI_s'\cos(\tfrac{\pi}{2}-\beta_x),$$

whence
$$W_1 = E\left[I_s'\cos\beta_x - I_s\cos\beta\right] = EI_p\left[\frac{\cos\beta_x}{K_{cx}} - \frac{\cos\beta}{K_c}\right],$$
$$W_2 = -E\left[I_s'\sin\beta_x - I_s\sin\beta\right] = -EI_p\left[\frac{\sin\beta_x}{K_{cx}} - \frac{\sin\beta}{K_c}\right].$$

Now β and β_x are small so that

$$\frac{1}{K_{cx}} - \frac{1}{K_c} \doteq \frac{W_1}{EI_p}$$

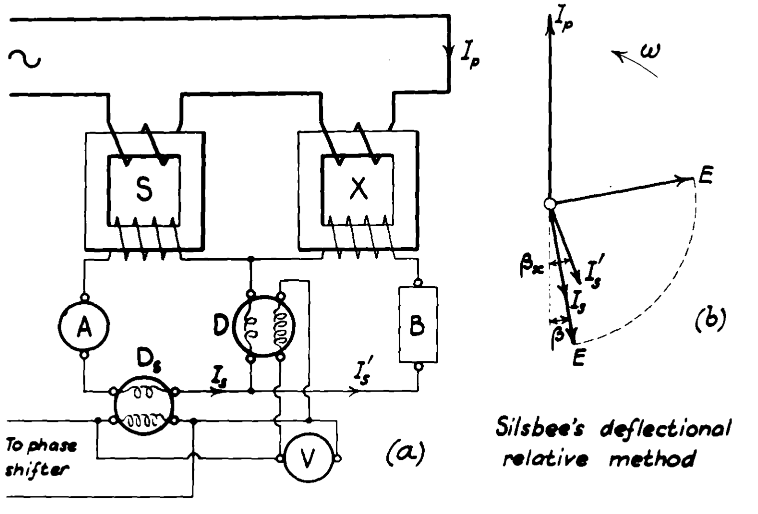
from which K_{cx} may be calculated. Again since $K_{cx} \doteq K_c$

$$\beta_x - \beta \doteq -\frac{W_2}{EI_p}$$

E is read on the voltmeter V ; I_p is calculated from and the observed maximum reading of D_p .

Silsbee* has described a method similar to the preceding in which the secondary currents from the two transformers are superposed/

* F.B.Silsbee, "A method of testing current transformers," Bull.Bur.Stds., vol.14, pp.317-329, 1919. The original suggestion appears to be due to W.A.Folger published in 1916 in the Proc. Pennsylvania Elec.Assoc., a journal not available in this country. See also F.B.Silsbee, loc.cit.ante., 1924; G.W.Stubbings, "Ratio and phase error tests on current transformers," Elec.Rev., vol.94, pp.604-606, 1924; F.A.Kartak, loc.cit.ante., 1920.



I_p
S
X
A
D
B
D_s
I_s
I_s'
V
To phase shifter
(a)
I_p
ω
E
β_x
I_s'
I_s
β
E
(b)
Silsbee's deflectional relative method

FIG. 40.

superposed in the current coil of a dynamometer D while the phase adjustment of the auxiliary voltage applied to the voltage coil is made by a second dynamometer D_S put into the secondary circuit of the standard transformer S, as in Fig.40a. The auxiliary voltage E, indicated by V, is put successively in phase and in quadrature with I_s and the readings of D observed in each case. If these in watts are W_1 and W_2, then from Fig.40b.

$$W_1 = E\left[I_s - I_s' \cos(\beta_x - \beta)\right],$$
$$W_2 = -E I_s' \cos\left(\tfrac{\pi}{2} - \beta_x + \beta\right) = -E I_s' \sin(\beta_x - \beta),$$

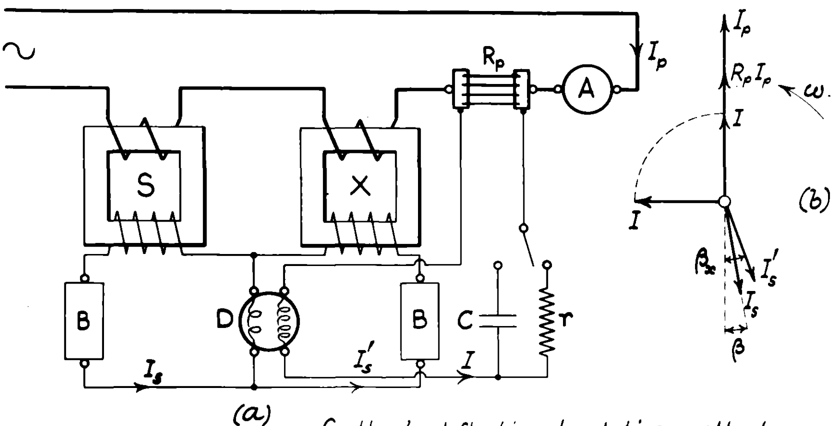
whence, $$\frac{K_{cx}}{K_c} \doteq \frac{1}{\left(1 - \frac{W_1}{EI_s}\right)} \doteq 1 + \frac{W_1}{EI_s},$$

and $$\tan(\beta_x - \beta) = \frac{-W_2/EI_s}{\left(1 - \frac{W_1}{EI_s}\right)} \doteq -\frac{W_2}{EI_s} \doteq \beta_x - \beta.$$

The current I_s may be read on the ammeter A, but is preferably computed from the maximum reading of D_S and the voltage E; indeed it must be so determined at low currents where the ammeter ceases to be sufficiently sensitive.

The detector D must have a current coil of low impedance and may conveniently be a wattmeter of 1 amp. range. Spilsbury* has recently designed a special portable dynamometer detector for test-room use scaled directly in current from +0.25 to -0.25 amp. at 55 or 110 volts on the volt coils. Readings on the scale give W_1/E and W_2/E directly and the centre zero enables/

* R.S.J.Spilsbury, "An instrument for workshop tests of current transformers," Journal Sci.Insts., vol.1, pp. 273-278, 1924.



Crother's deflectional relative method

FIG. 41.

enables their sign to be correctly taken.

In general the ratio and phase errors of X will be greater than those of S ; to determine this point an additional resistance should be added to the burden of X . Then if the ratio and phase errors of X were originally greater than those of S the readings W_1 and W_2 will be increased; if less, the readings will be reduced or even reversed.

3. Single dynamometer Method.

Crothers[*] has introduced a simple modification of the methods of Barbagelata and Silsbee described in Section 2, illustrated in Fig.41a. In this a single dynamometer is used, its volt coil being of negligible reactance. Neglecting the shunting effect of the high resistance voltage circuit upon the four-terminal resistance R_p the reading of D in watts will be, very closely,

$$\begin{aligned} W_1 &= R_p I_p \left[I_s \cos(\pi-\beta) - I_s' \cos(\pi-\beta_x) \right] \\ &= R_p I_p \left[I_s' \cos\beta_x - I_s \cos\beta \right] \end{aligned}$$

as Fig.41b shows. From this,

$$\frac{1}{K_{cx}} - \frac{1}{K_c} \doteqdot \frac{W_1}{R_p I_p^2}$$

The method so far resembles Barbagelata's procedure and gives the ratio error of X ; Crothers does not state how the phase error may be found. If this diagram is compared with that for Drysdale's absolute method for a 1/1 ratio transformer, Fig.20a, it will be seen that the two are very similar. In

Drysdale's/

* H.M.Crothers, "Field testing of instrument transformers," Elec.World, vol.74, pp.119-121, 1919.

Drysdale's method the approximately equal primary and secondary currents of a 1/1 transformer are superposed in the dynamometer coil; in the present method the superposed, nearly equal, currents are obtained from two separate transformer secondaries. This suggests that the phase error may be found by substituting for the resistance in the volt circuit of D a condenser of equal reactance; the current then advances in phase on I_p by 90° and the new reading of the dynamometer will be

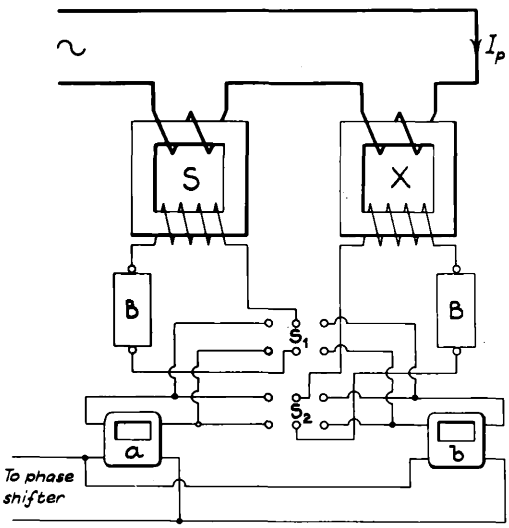
$$W_2 = R_p I_p \left[I_s \cos\left(\tfrac{3\pi}{2} - \beta\right) - I_s' \cos\left(\tfrac{3\pi}{2} - \beta_x\right)\right]$$
$$= -R_p I_p \left[I_s \sin\beta - I_s' \sin\beta_x \right]$$

whence $\beta_x - \beta \doteq \dfrac{W_2 K_c}{R_p I_p^2}$

The method is suggested as very convenient for testing transformers on site since the amount of apparatus required is slight and of a readily portable nature.

4. Agnew's watt-hour meter method.

The two ammeter method of Section 1 of the present Chapter may be readily modified so as to obtain superior precision, and at the same time determine the phase-angle error, by substituting the current coils of two separately excited dynamometers for the ammeters. The ratio error can be found by adjusting the phase of the volt coil excitation until each dynamometer in turn gives a maximum reading. A second/



Agnew's watt-hour meter relative method

FIG. 42.

second pair of readings with the auxiliary voltage adjusted approximately 90° from the maximum position enables the phase shifter be found; or alternatively the angle through which the phase shifter is to be moved to bring each dynamometer separately to zero can be observed, this angle being the phase difference between the two secondary currents. Calibration errors in the dynamometers can be eliminated by repeating the observations with the instruments interchanged in the secondary circuits.

Since the differences in ratio and phase-angle of the two transformers are usually small, the interchanged dynamometer method is not capable of very high precision. Agnew* has shown, however, that if watt-hour meters be substituted for dynamometers, as in Fig.42, considerable precision can be attained, since the effects of differences in ratio and phase-angle are cumulative in such instruments. The meters a and b are of similar type and should first be carefully adjusted to work correctly on full current at unity and zero power factor. It is a great convenience if the ratio of gearing can be arranged so that 10 revolutions of the disc correspond with one division on the first dial; the discs may also be marked off with equally spaced divisions to enable fractions of a turn to be observed. It is an advantage if the meters be speeded up above the normal running speed; this can/

* P.G.Agnew, "A watthour meter method of testing instrument transformers," Bull.Bur.Stds., vol.11,pp.347-357, 1915. Also F.A.Kartak, loc.cit.,1920; A.Barbagelata, loc.cit., 1921; F.B.Silsbee, loc.cit.,1924. An identical process is described by A.Alemany, "Note sur une methode pratique de vérification des transformateurs de mesure," Rev.Gen.de l'El.,vol.18,pp.515-518, 1925.

can be arranged by shunting the brake magnets with small iron bridge pieces. Observations are made of the number of revolutions made by each meter disc (i) with S_1 to the left and S_2 to the right and (ii) with S_1 to the right and S_2 to the left, both at a known power factor between the auxiliary voltage and I_p. Let a_s, b_x be the numbers of revolutions with arrangement (i) and a_x, b_s the corresponding revolutions with (ii) each in the same total time. If the meters are designed so that the dial constant k, the nominal watt-hours per rev. is the same for each, let m_a, m_b be the rates of the two meters, i.e., the ratio of recorded watt-hours to true watt-hours. Then if, in general, a meter is connected to a circuit of power-factor $\cos\phi$ via a current transformer of ratio K_c and phase-angle β the watt-hours in the circuit will be $\dfrac{K_c \cos\phi}{\cos(\phi-\beta)} \cdot \dfrac{kr}{m}$ for r revolutions of the disc. This can be written as $\dfrac{K_c\, k\, r}{m \cos\beta\,(1+\tan\phi\tan\beta)}$. Applying this formula to condition (i)

$$\frac{K_c\, k\, a_s}{m_a \cos\beta\,(1+\tan\phi\tan\beta)} = \frac{K_{cx}\, k\, b_x}{m_b \cos\beta_x\,(1+\tan\phi\tan\beta_x)}$$

and to (ii)

$$\frac{K_{cx}\, k\, a_x}{m_a \cos\beta_x\,(1+\tan\phi\tan\beta_x)} = \frac{K_c\, k\, b_s}{m_b \cos\beta\,(1+\tan\phi\tan\beta)}$$

at any power factor whatever. Now $\cos\beta \doteqdot \cos\beta_x \doteqdot 1$ since the angles are small. If a_s, b_x, a_x, b_s are taken with $\phi = 0$, i.e., with the auxiliary voltage and I_p in phase then

$$\frac{K_c\, a_s}{m_a} \doteqdot \frac{K_{cx}\, b_x}{m_b},$$

and

$$\frac{K_{cx}\, a_x}{m_a} \doteqdot \frac{K_c\, b_s}{m_b},$$

whence,

$$\frac{K_{cx}}{K_c} \doteq \sqrt{\frac{a_s}{a_x} \cdot \frac{b_s}{b_x}}$$

If $\cos\phi$ be now altered to be approximately 0.5 or less so that the meters work on low power-factor, let a_s', b_x', a_x', b_s' be the new readings with conditions (i) and (ii), then

$$\frac{a_s'}{a_x'} \cdot \frac{K_c}{K_{cx}} \cdot \frac{(1+\tan\phi\tan\beta_x)}{(1+\tan\phi\tan\beta)} \doteq \frac{b_x'}{b_s'} \cdot \frac{K_{cx}}{K_c} \cdot \frac{(1+\tan\phi\tan\beta)}{(1+\tan\phi\tan\beta_x)}$$

or

$$\frac{(1+\tan\phi\tan\beta_x)^2}{(1+\tan\phi\tan\beta)^2} \doteq \frac{a_x' b_x'}{a_s' b_s'} \cdot \frac{K_{cx}^2}{K_c^2}$$

Remembering that $\tan\beta$ and $\tan\beta_x$ are very small,

$$1 + 2\tan\phi\,(\tan\beta_x - \tan\beta) \doteq \frac{a_x' b_x'}{a_s' b_s'} \cdot \frac{K_{cx}^2}{K_c^2}$$

whence

$$\tan\beta_x - \tan\beta \doteq \beta_x - \beta \doteq -\frac{1}{2\tan\phi}\left[1 - \frac{a_x' b_x'}{a_s' b_s'} \cdot \frac{a_s b_s}{a_x b_x}\right]$$

from which the angle error is found.

The signs in the above expressions are correct for both secondary currents leading on $-I_p$ and $\beta_x > \beta$. To test which transformer has the greater errors, add a further non-inductive load to secondary of X. If the difference is increased then X had originally greater errors than S.

Agnew shows that by taking a sufficiently great number of revolutions ample accuracy can be attained in all commercial tests/

tests; the ratio can be found to 0.03% and the angle within 1 or 2 minutes. The method is independent of line fluctuations and requires no specialised apparatus; it is, therefore, admirably suited to tests on site. Simplified procedure for this purpose has been suggested by Crothers* and by Craighead and Weller.† The last named investigators reduce the time of a test by a carefully planned procedure and the use of a tabular form of calculation; 7 hours is taken for a 6 point check. The standard transformer should be carefully calibrated and not differ more than 5% in ratio or 1° in angle from the unknown. The meters must be well adjusted and their difference in rate must be in the same direction for $\cos\phi = 1$ as for $\cos\phi = 0.5$; they must not creep. The following results show the order of agreement obtained on a 10/5 amp. transformer at 60 cycles.

Primary current.	Absolute method. Ratio factor	β	Watthr. meter method Ratio factor	β
1	1.0017	+29 min.	1.0010	+34 min.
2	0.9997	21	0.9996	23
4	0.9977	14	0.9977	18
6	0.9971	12	0.9969	13
8	0.9967	10	-	-
10	0.9959	9	0.9961	12

* H.M.Crothers, Elec.World, Vol.74, pp.119-121, 1919.

† J.R.Craighead and C.T.Weller, "Watthour meter method of testing current transformers for ratio and phase-angle," Gen.Elec.Rev., vol.24, pp.542-551, 1923.

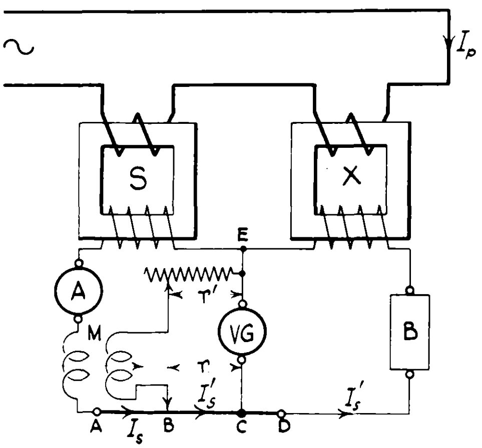
The secondary burden in the above test was 0.396 ohm ~~and~~ and 510 μH, or 11 voltamperes at 0.9 power-factor for 5 amp., 60 cycles.

In Section 4 of Chapter III of the present part an absolute method for testing a 1/1 transformer by the use of two watthour meters is described. Comparing Fig.22a with Fig.42 it is easy to see that the absolute method is equivalent to testing by Agnew's method an unknown transformer (the secondary current) with a perfect 1/1 transformer (the primary current). If the meters be interchanged to eliminate differences in calibration, putting $K_{cx} = 1$, $\tan\beta_x = 0$ gives for the ratio and phase angle

$$K_c \doteq \sqrt{\frac{a_p b_p}{a_s b_s}}$$

$$\tan\beta \doteq \frac{1}{2\tan\phi}\left[1 - \frac{a'_p b'_p}{a'_s b'_s}\cdot\frac{a_s b_s}{a_p b_p}\right]$$

where a_p, b_p, a_s, b_s are the readings of meters a and b when in the primary and secondary circuits respectively.



Silsbee's null relative method

FIG. 43

CHAPTER VI.

RELATIVE METHODS. NULL.

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1. Silsbee's bridge method.

Silsbee has shown* that the deflectional method of Fig.40a can be readily adapted for null operation by the arrangement shown in Fig.43. AD is a slide wire of 0.2 ohm resistance; r' a resistance of 30 ohms adjustable in steps of 2 ohms; M is a variable mutual inductor of about 600 microhenry inductance. By regulation of M and r the vibration galvanometer can be made to give zero reading; then E and C are at the same potential. The vector difference between the secondary currents I_s' and I_s will then circulate through the path EB. If the resistance of this path be R ohms and L is the inductance of the secondary winding of M, let i_s' and i_s be harmonic vectors of the secondary currents. Then equating the p.d. between E and C to zero, assuming B to the left of C gives

$$(R+j\omega L)(i_s'-i_s)+ri_s'-j\omega Mi_s=0$$

or

$$(R+r+j\omega L)i_s' = [R+j\omega(L+M)]i_s,$$

whence

$$\frac{i_s}{i_s'} = \frac{R+r+j\omega L}{R+j\omega(L+M)} = \frac{1+\frac{r}{R}+j\omega\frac{L}{R}}{1+j\omega\left(\frac{L}{R}+\frac{M}{R}\right)}.$$

Now for brevity write

$$a=r/R,\quad b=\omega M/R,\quad c=\omega L/R$$

and/

* F.B.Silsbee, Bull.Bur.Stds., vol.14, pp. 317-329, 1919; also see F.B.Silsbee, loc.cit.ante, 1924.

and remember that these quantities are all small; hence to the second order of small quantities,

$$\frac{i_s}{i'_s} = \frac{1+a+jc}{1+j(b+c)} \doteq (1+a-b^2-bc) - j(b+ab+ac)$$

Now in terms of the primary current

$$i_s = \frac{I_p}{K_c}\,\epsilon^{-j(\pi-\beta)} \quad \text{and} \quad i'_s = \frac{I_p}{K_{cx}}\,\epsilon^{-j(\pi-\beta_x)}$$

whence

$$\frac{i_s}{i'_s} = \frac{K_{cx}}{K_c}\,\epsilon^{-j(\beta_x-\beta)} = \frac{K_{cx}}{K_c}\left[\cos(\beta_x-\beta) - j\sin(\beta_x-\beta)\right]$$

Comparing the expressions

$$\frac{K_{cx}}{K_c}\cos(\beta_x-\beta) = 1+a-b^2-bc$$

$$\frac{K_{cx}}{K_c}\sin(\beta_x-\beta) = b+ab+ac$$

Solving these equations, and again retaining only terms of the first and second order,

$$\frac{K_{cx}}{K_c} = 1+a-\frac{b^2}{2}-bc$$

$$\tan(\beta_x-\beta) = b+ac$$

Substituting,

$$K_{cx} = \left[1+\frac{r}{R}-\frac{\omega^2M^2}{2R^2}-\frac{\omega^2ML}{R^2}\right]K_c\,,$$

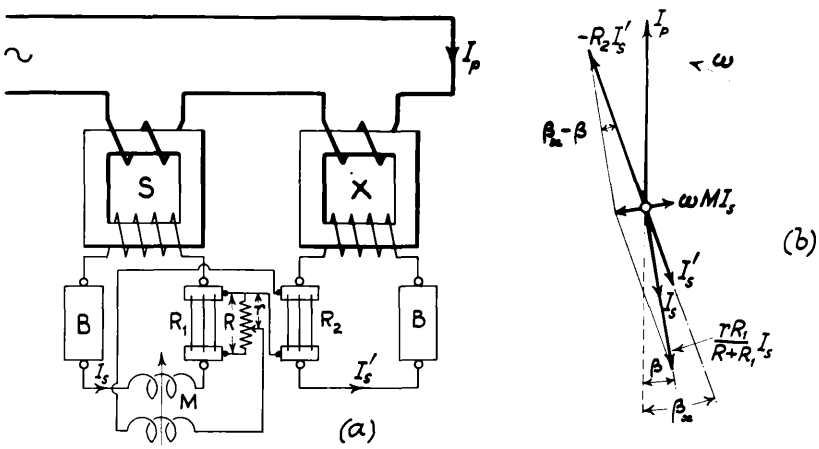
$$\tan(\beta_x-\beta) = \frac{\omega M}{R}+\frac{\omega Lr}{R^2}.$$

If it is necessary to move B to the right of C for balance then,

$$K_{cx} = \left[1-\frac{r}{R}+\frac{r^2}{R^2}-\frac{\omega^2M^2}{2R^2}-\frac{\omega^2ML}{R^2}\right]K_c\,,$$

$$\tan(\beta_x-\beta) = \frac{\omega M}{R}-\frac{\omega Lr}{R^2}-\frac{\omega Mr}{R^2}.$$

These/



Barbagelata's null relative method

FIG. 44.

These four equations hold correctly if $\beta_x > \beta$. By arranging S to have always the smaller ratio CD may be omitted entirely. To determine which transformer has the greater phase-angle, if B is to left of C, so that S supplies the greater current and has the smaller ratio, add resistance to the secondary of X. Then the lead of I_s' on I_p reversed will increase; if M is increased to re-balance then I_s' was originally leading more than I_s and M is to be taken as positive in the first pair of equations. As a rule, unless the two transformers are of widely different characteristics none but the first order terms need be taken into account; then very nearly,

$$K_{cx} \doteq \left(1+\frac{r}{R}\right)K_c$$
$$\tan(\beta_x-\beta) \doteq \frac{\omega M}{R} \doteq \beta_x-\beta$$

with B to the left.

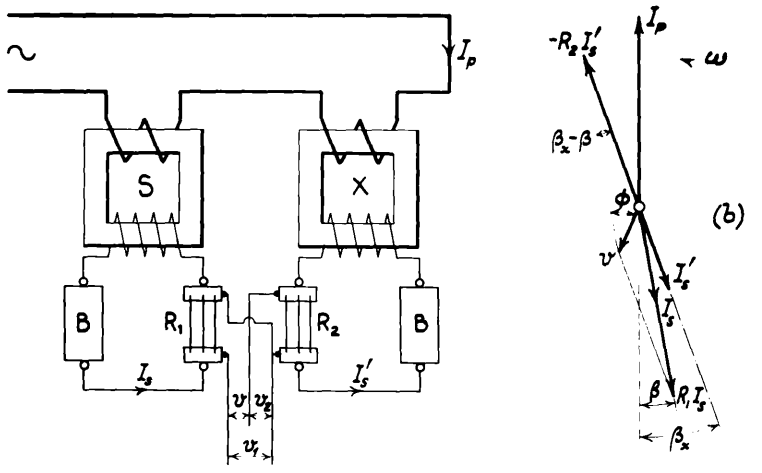
2. <u>Barbagelata's bridge method</u>.

This simple method,[*] Fig. 44a, is primarily intended for comparison of transformers of unequal ratio. If S has a greater ratio than X, $I_s R_1$ is made greater than $I_s' R_2$. Balance is attained by adjustment of r and M; then the drop of voltage in r and the e.m.f. of mutual induction exactly balance $R_2 I_s'$, as shown in the vector diagram of Fig.44b. Then

$$R_2 I_s' \cos(\beta_x-\beta) = \frac{r R_1}{R+R_1} I_s$$
$$R_2 I_s' \sin(\beta_x-\beta) = \omega M I_s$$

From/

[*] A.Barbagelata, <u>loc.cit.</u>, 1921.



Use of A.C. potentiometer for relative tests

FIG. 45.

From the first,

$$\frac{I_p}{I_s'} = \frac{R_2}{R_1} \cdot \frac{R+R_1}{T} \cdot \frac{I_p}{I_s} \cos(\beta_x - \beta)$$

or,

$$K_{cx} \doteq \frac{R_2}{R_1} \cdot \frac{R+R_1}{T} \cdot K_c \doteq \frac{R_2}{R_1} \cdot \frac{R}{T} \cdot K_c$$

since β_x and β are small and R_1 can be neglected in comparison with R. Again

$$\tan(\beta_x - \beta) = \frac{\omega M}{T R_1}(R+R_1) \doteq \frac{\omega M R}{T R_1}$$

The method is easy to operate and is of wide range; it enables a single standard transformer to be used to test others of any desired ratio.

3. A.C.Potentiometer Method.

Two transformers of any ratio, equal or unequal, can be compared* by including a suitable four-terminal resistance in each secondary circuit, as in Fig.45a, and measuring by means of an a.c. potentiometer the volt drops v_1, v_2 across these resistances and their vector difference v, as well as the phase displacement ϕ between v and v_1. Then

$$K_{cx} = \frac{R_2}{R_1} \cdot \frac{v_1}{v_2} \cdot K_c$$

Also from the vector triangle of Fig.45b,

$$v_2 \sin(\beta_x - \beta) = v \sin\phi$$

or $$\sin(\beta_x - \beta) = \frac{v}{v_2} \sin\phi \doteq \beta_x - \beta.$$

The method is very convenient when a suitable potentiometer is available, and is capable of considerable precision.

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* D.C.Gall, Elecn., vol. 83, pp. 603-604, 1920.

PART III.

TESTING OF VOLTAGE TRANSFORMERS.

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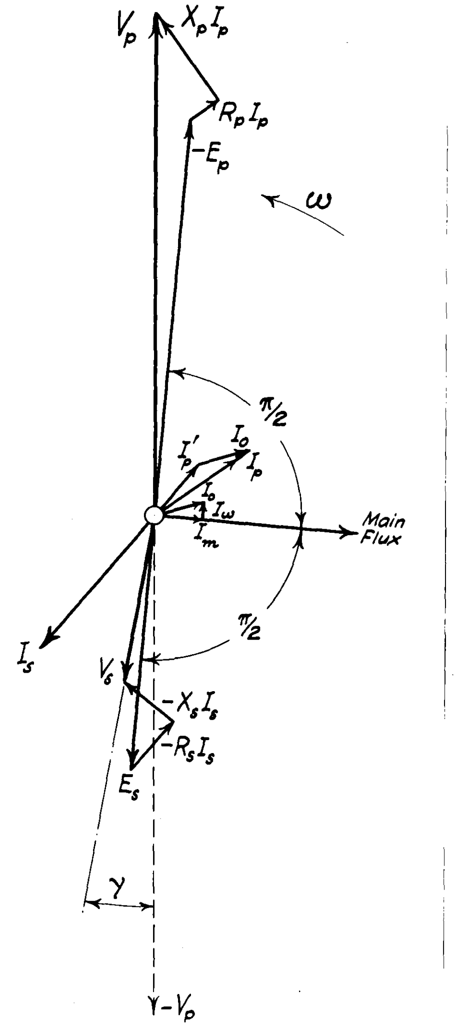


V_p
$X_p I_p$
$R_p I_p$
$-E_p$
ω
$\pi/2$
I_0
I_p'
I_p
I_o
I_ω
I_m
Main Flux
$\pi/2$
I_s
V_s
$-X_s I_s$
$-R_s I_s$
E_s
γ
$-V_p$

FIG. 46

CHAPTER I.

INTRODUCTORY.

1. Theory of the voltage transformer.

The vector diagram of the voltage transformer, shown in Fig.46, involves a little more detail than that of the current transformer, Fig.9. Starting in the customary way with the main flux in the core, linked mutually with both primary and secondary windings, an e.m.f. E_s will be induced thereby in the secondary, E_s lagging by a quarter period behind the flux. This e.m.f. causes a current I_s to circulate in the secondary circuit, the magnitude and phase of which depend upon the resistance and reactance of the secondary winding together with that of the external burden with which the transformer is loaded. Deducting from E_s the resistance and leakage reactance drops, $R_s I_s$ and $X_s I_s$ through the secondary winding gives V_s the p.d. at the terminals of the burden. Turning now to the primary side, the main flux will induce in the primary winding a back e.m.f. E_p in phase with E_s and such that $E_p/E_s = T_p/T_s$; the primary applied voltage must then have a component $-E_p$ to balance this induced voltage. The primary current is composed of two principal components I_o and I_p' . The former I_o is necessary to provide a magnetising component I_m in phase with and responsible for the production of the main flux, and an iron loss component I_w in phase with $-E_p$ accounting for the power dissipated by eddy currents and hysteresis. The second component I_p' is required to balance the secondary ampere-turns, is opposite in phase to I_s and/

and of such magnitude that $I_p' T_p = I_s T_s$. I_p is the vector sum of I_p' and I_o. To obtain the voltage V_p that must be applied to the terminals of the primary winding, it is necessary to add to $-E_p$ the resistance drop $R_p I_p$ and the leakage reactance drop $X_p I_p$ in that winding, as shown in the diagram. The ratio of the transformer is then $K_V = V_p / V_s$ and the phase-angle is γ, the angle between V_s and V_p reversed. For the reason explained in Section 6 of Chapter I, Part I, γ is taken as positive when V_s lags on $-V_p$. It should be observed that V_s is always less than E_s, except at no-load when they are equal; moreover under ordinary conditions V_p is greater than $-E_p$. Hence the ratio K_V is always more than the ratio of turns T_p/T_s.

It should be clearly realised that the conditions under which the voltage transformer works are quite different from those imposed upon current transformers. In the latter the primary current, and hence the primary applied voltage, is varied over a wide range, with the consequent effects on ratio and phase-angle that have been discussed in the preceding Part. In the voltage transformer, however, the primary voltage is practically constant under normal conditions; interest centres, therefore, upon the way in which the characteristics depend on the volt-amperes taken from the secondary, <u>i.e.</u>, on the number and nature of the instruments forming the secondary burden. In this respect the voltage transformer presents a problem essentially similar to that provided by the power transformer and simpler than that of the current transformer. Since the primary voltage is/

is constant so also is the flux, no matter what may be the secondary load; the exciting current I_0 is therefore the same from no-load to full load and it would be expected, therefore, that the change in ratio with burden would be practically independent of I_0 . Experiment and theory* show this to be the case. The change in ratio for various secondary burdens must depend, therefore, almost entirely on the resistances and leakage reactances of the windings of the transformer. Since the leakage can be kept small by interleaving the primary and secondary coils or by a properly designed concentric construction, it follows that the change in ratio and angle both with primary voltage and with frequency will be small. Hence it is further to be expected that the voltage transformer is a piece of apparatus of very high precision. It must not be supposed, however, that because the characteristics are practically independent of the exciting current that this quantity can be disregarded. It is essential, just as in current transformers, to keep I_0 very small by using a short magnetic circuit of good iron of high permeability and low loss; otherwise the changes of I_0 with voltage and frequency may become sufficient to affect the values of K_V and γ to an appreciable extent.

It is possible to develop expressions for K_V and γ in just the same way as was done for the values of K_C and β in the theory of the current transformer; these expressions are rather less simple and, for the present object, not of very great/

* M.G.Lloyd and P.G.Agnew, "The regulation of potential transformers and the magnetising current," Bull.Bur.Stds., Vol.6, pp. 273-280, 1910.

great value. The indirect method of testing,* although very occasionally used, is not really required for voltage transformers since the direct methods are simple and of wide application. Space will not be devoted, therefore, to development of the analytical expressions upon which the indirect method depends. Those interested will find the matter fully dealt with in Lloyd and Agnew's paper just cited.†

2. Typical values of ratio and phase-angle.

The curves of Fig.47 show typical ratio and phase-angle values plotted from the classic results of Agnew and Fitch. The curves relate to a transformer of 2200/110 volt ratio, 50 volt-ampere output, with R_p = 460 ohms and R_s = 0.393 ohms. The normal frequency is 50 cycles per second and the tests were made with a non-inductive burden. The isolated points R, P are for an inductive burden of power-factor 0.2.

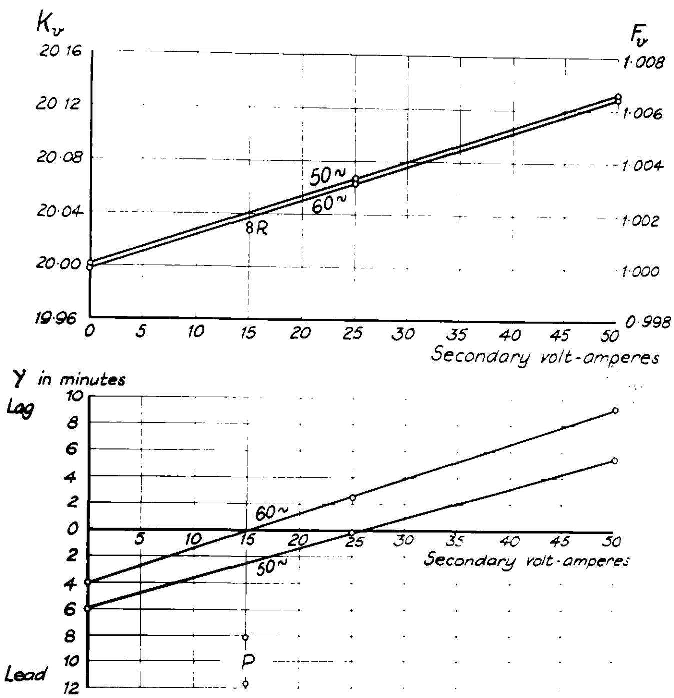
It will be seen that the change in ratio from no-load to full-load is less than 0.7%; in very good transformers it may be as low as 0.3% but it is invariably less than 1%. The phase-angle is never more than about 10 minutes, and is for most practical purposes quite negligible.

Fig. 47a shows that both ratio and phase-angle curves are linear functions of the secondary volt-amperes at normal, constant primary voltage.

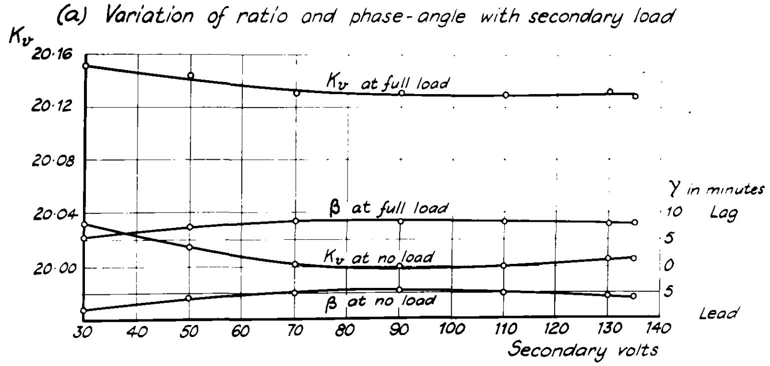
3./

* M.Iliovici, "Methode d'essai des transformateurs de mesure de tension," Lum.Elect., vol.33, pp.276-277, 1916; Bull.Soc.Int. des. Elecns., vol.6, pp.155-187, 1916.

† See also F.A.Laws, Electrical Measurements, pp.575-576, 1917.



(a) Variation of ratio and phase-angle with secondary load



(b) Variation of ratio and phase-angle with secondary voltage

FIG. 47.

3. Variation of characteristics with working conditions.

It is now necessary to examine briefly the variations in the characteristics with various changes in the working conditions. These are (a) frequency, (b) secondary burden, at constant primary voltage; and (c) secondary (or primary) voltage, at constant load. Also there are the unimportant questions of (d) wave-form and wave distortion.

3a. Frequency. - The general effect of a reduction of frequency, with the consequent increase in flux and I_0 at constant primary voltage, is the same as in a current transformer, namely, to increase both ratio and phase-angle; see Fig.47a. The magnitude of the increase is, however, very much less than in the current transformer.

3b. Secondary burden. - The variation of K_V and γ with the amount of secondary burden of a given power-factor is, as mentioned above, strictly linear. In some cases, as when the highly inductive voltage coil of an induction type watt-hour meter forms the burden, the secondary power-factor may become very low. The effect of this is, as shown by the points R, P in Fig.47a, to reduce the ratio slightly and to increase fairly considerably the phase-angle. Hence, as with a current transformer, it is essential to specify not merely the volt-amperes absorbed by the burden but also its time-constant if the figures and angle are to be of practical value.

3c/

3c. Voltage. - If the voltage at which the transformer is worked be changed, the frequency being fixed, variations of K_V and γ are found which are represented by the curves of Fig.47b, showing K_V and γ as functions of the secondary volts, both at no-load and full-load. The general tendency of a reduction of voltage is to increase both ratio and phase-angle, but the changes are very slight over a wide range of voltage variation.

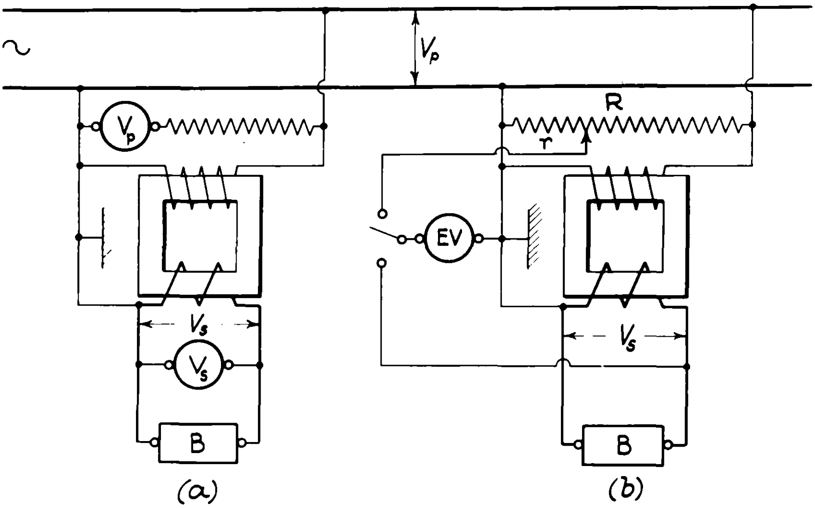
3d. Wave-form and wave distortion. - As in the case of the current transformer the influence of wave-form on the values of K_V and γ is quite negligible. Further, the distortion introduced by the transformer is inappreciable; the secondary voltage has, therefore, a wave-form identical with that of the primary voltage.

4. Introduction to methods of testing.

The indirect method of testing voltage transformers has occasionally been used; the direct methods are, however, so simple and of such superior accuracy that they are now invariably employed. In principle the indirect test resembles the well-known short circuit test applied in the measurement of regulation of power transformers.

As with the current transformers the Direct methods of testing voltage transformers may be either Absolute or Relative. The Absolute methods are all based upon the principle of comparing a known fraction of the primary voltage with that of the secondary both for magnitude and phase displacement; the comparison may be made, by means of suitable separately excited dynamometers/

dynamometers or electrometers, by a deflectional method. Alternatively a null method can be devised by the inclusion of a suitable compensating circuit. All absolute methods essentially necessitate the connection of some part of the testing apparatus into the high voltage or primary side; this may be entirely avoided by utilising deflectional or null relative methods in which two transformers are compared by measurements made entirely upon their secondary circuits. Each of these four classes of methods will be described in the following chapters.



Voltmeter absolute methods

FIG. 48.

CHAPTER II.

Absolute Methods. Deflectional.

1. Two voltmeter methods.

The ratio of a voltage transformer is most easily determined with the aid of two voltmeters,* as shown in Fig.48a, one connected in parallel with the primary winding and the other with the secondary winding. The two sides of the transformer are joined at one point, which is earthed for safety. Then if V_p and V_s be the readings of the voltmeters,

$$K_V = \frac{V_p}{V_s} ;$$

the instruments must necessarily be calibrated.

The necessity for calibration may be avoided by an artifice due to Laws.† Two similar voltmeters are used, connected first as in Fig.48a, the resistance in series with the primary voltmeter being adjusted until the reading v_p on it is about equal to the reading of the instrument on the secondary side v_s. The two voltmeters are then removed, connected in series, and a current I passed through them so that they give readings v_p', v_s' about equal to the preceding values. Then clearly, if R_p be the total resistance of the primary voltmeter and its series resistance, R_s the resistance of the secondary voltmeter,

$$V_p = R_p \frac{v_p}{v_p'} I , \qquad V_s = R_s \frac{v_s}{v_s'} I,$$

---

* F.A.Kartak, loc.cit.ante, 1920.

† F.A.Laws, Electrical Measurements, p.584, 1917.

whence,

$$K_v = \frac{R_p}{R_s} \cdot \frac{v_p}{v_p'} \cdot \frac{v_s'}{v_s}$$

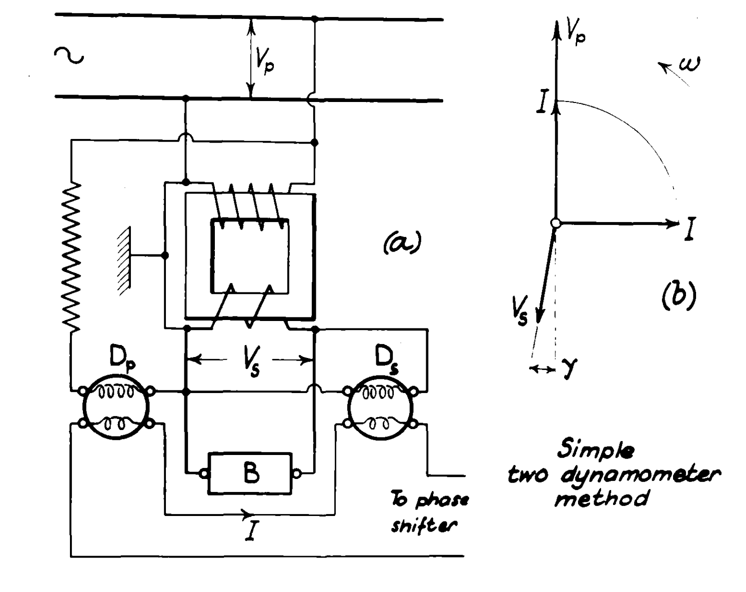
A further variant of the ~~two~~ voltmeter method is that shown in Fig.48b. Here a single electrostatic voltmeter is connected successively across the secondary terminals and then across a portion r of a potential divider resistance R in parallel with the primary; r is adjusted until the reading v_p of the voltmeter is about the same as the reading v_s when it is put across the secondary. By this means calibration of the voltmeter is obviated and thus since $V_p = \frac{R}{r} v_p$, $V_s = v_s$,

$$K_v = \frac{R}{r} \cdot \frac{v_p}{v_s}.$$

The disadvantage of the two voltmeter method is the fact that it only gives the ratio of the transformer. Moreover, the possible accuracy is not very high and the sensitiveness falls off considerably at lower readings on the voltmeter scales owing to their non-uniformity.

2. Two dynamometer methods.

The sensitivity of the two voltmeter method can be increased and, at the same time, both ratio and phase-angle be determined by the substitution of separately excited dynamometers for the ordinary types of voltmeter. Referring to Fig.49a the voltage circuits of two dynamometers, D_p and D_s , are connected respectively to the primary and secondary terminals of the transformer. The fixed coils of the dynamometers are connected in/



V_p
V_s
D_p
D_s
B
I
(a)
To phase shifter
V_p
I
I
ω
V_s
γ
(b)
Simple two dynamometer method

FIG. 49

in series and carry a current I supplied by a phase shifting device.* The phase shifter is first regulated until D_p gives a maximum reading, see Fig. 49b; then if the volt circuit of this dynamometer is non-reactive V_p and I are in phase and the reading of D_p will be, in watts,

$$W_p = V_p I$$

while the reading of D_s with the same supposition of a non-reactive volt circuit will be

$$W_s = V_s I \cos(\pi - \gamma) = -V_s I \cos\gamma.$$

The phase of I is now rotated through 90°, making D_p read zero; D_s will then give a new reading of

$$W_s' = V_s I \cos\left(\frac{3\pi}{2} - \gamma\right) = -V_s I \sin\gamma.$$

Then, the numerical value of the ratio is, very nearly,

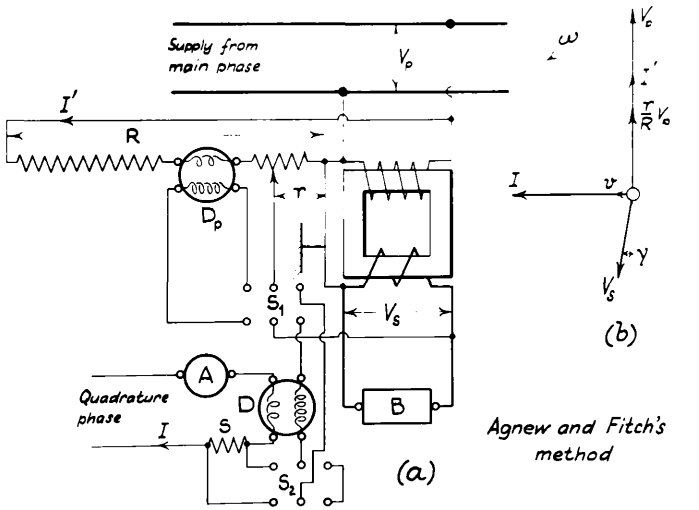
$$K_v \doteq \frac{W_p}{W_s}$$

since $\cos\gamma \doteq 1$; the angle is given by

$$\tan\gamma = \frac{W_s'}{W_s} \doteq \gamma.$$

The method is capable of considerable precision and is of wide range. The main source of error lies in the supposition that the voltage circuits of the dynamometers are non-reactive. The error is easily calculated, in the manner already employed in the theory of the two dynamometer method of testing current transformers in Part II, Chapter III, Section 2. Just as in that case the error will be zero either if the volt-circuits are of equal time-constant or alternatively if their inductance be suitably

* A. Barbagelata, loc. cit. ante., 1921; F. A. Kartak, loc. cit. ante., 1920. The process was used by L. T. Robinson, loc. cit. 1910, for measurement of γ. A full test room set-up is described in his paper.



Supply from
main phase
V_p
I'
R
D_p
r
S_1
V_s
A
Quadrature
phase
D
S
I
B
S_2
(a)
V_c
$\frac{r}{R}V_o$
I
v
γ
V_s
(b)
Agnew and Fitch's
method

FIG. 50.

suitably compensated by the well-known shunted condenser device so often used in wattmeters.

A number of methods, essentially the same in principle, have been described in which a two-phase or a three-phase source of supply is utilised instead of the phase shifting device for exciting the dynamometers. Of these the most important is the method of Agnew and Fitch,* shown in Fig.50a, and used by these experimenters in tests of the highest accuracy. The transformer is supplied from the main phase of a two-phase alternator through a step-up transformer, and across its primary a high resistance R is connected. In series with R is the fixed coil of a dynamometer D_p. The voltage drop in a fraction r/R ~~and~~ of the resistance is opposed to the secondary voltage of the transformer at the switch S_1. With S_1 to the left r is adjusted until D_p reads zero; then the current I' and the resultant voltage applied to the volt-coil of D_p, assumed non-reactive, are in quadrature. Neglecting the reactance of R, I' is in phase with V_p; further neglecting the shunting effect of the volt coil of D_p on r the resultant voltage applied thereto is the vector sum of V_s and $\frac{r}{R}V_p$, namely v in Fig.50b. S_1 is now thrown to the right so that when S_2 is also to the right v is applied to the volt coil of a dynamometer D, the current coil of which is supplied with a current I from the second or quadrature phase of the alternator. Since I is in phase with v, the reading of D is a measure of v, and the instrument/

* P.G.Agnew and T.T.Fitch, Bull.Bur.Stds., vol.6, p.281, 1910.

instrument may be calibrated as a voltmeter. It should be observed that since the resistances of the transformer windings are small in comparison with R the volt coil of D is practically connected in series with the parallel combination of r and $R-r$, as can be readily verified from the diagram. Calibration under the conditions of test is easily effected by disconnecting the primary from the main phase, throwing S_2 to the left, thus applying a known voltage SI to the volt coil of D while still connected to the potential divider.

From the approximate vector diagram it is obvious that

$$V_s \cos\gamma = \frac{r}{R} V_p$$

$$V_s \sin\gamma = v$$

whence,

$$K_v = \frac{R}{r}\cos\gamma \doteq \frac{R}{r}$$

$$\sin\gamma = \frac{v}{V_s} \doteq \gamma$$

V_s can be measured by a suitable voltmeter included in the secondary burden.

Possible sources of error are due to residual inductance in r, $R-r$ and the voltage circuit of D. The influence of the latter on both ratio and phase-angle is quite negligible. The effect of residuals in r and $R-r$ on the ratio is less than 1 part in 50,000 in the most unfavourable case. In the case of the phase-angle Agnew and Fitch show that

$$\gamma \doteq \frac{v}{V_s} + \left(\frac{\omega L_r}{r} - \frac{\omega L_R}{R}\right),$$

the bracketed correction being the difference between the time-constants of r and R. Hence if these time-constants be equal, residuals/

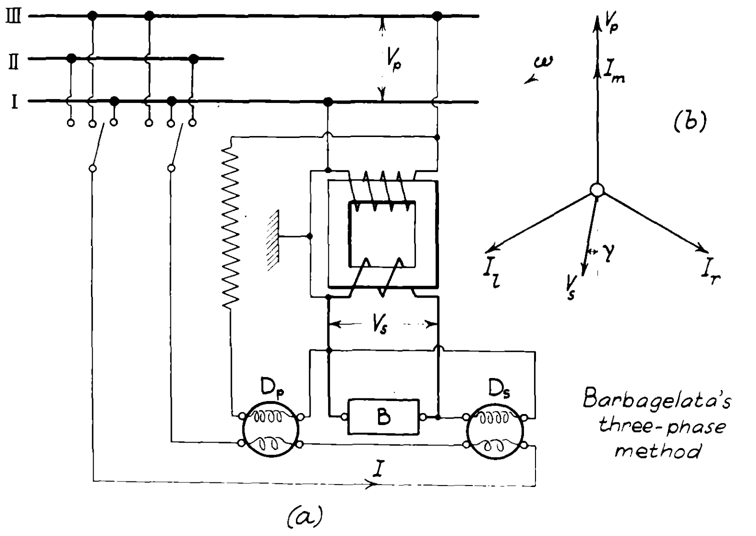

III
II
I
V_p
ω
V_s
D_p
D_s
B
I
(a)
V_p
I_m
(b)
I_l
V_s
γ
I_r
Barbagelata's three-phase method

FIG. 51

residuals introduce no error. It is easy in practice to construct resistances giving a correction of less than 1 minute at commercial frequencies. The instruments used by these workers enabled a sensitivity of 1 part in 20,000 in ratio and 0.1 minute in phase-angle to be attained.

Of three-phase methods the most important is that introduced by Barbagelata* illustrated in Fig.51a and corresponding with the method for current transformers shown in Fig.17a. The test transformer is supplied from lines III and I of a three-phase system through a step-up transformer not shown in the diagram. The volt coils of two dynamometers D_p, D_s are used as voltmeters across the primary and secondary windings respectively, as in Fig.49a; the current coils are connected in series and can be put at will across lines III and I, I and II, II and III by turning the switches successively to the middle, right and left contacts. In each position of the switches equal currents, of magnitude I, will be passed through the current coils respectively in phase, 120° behind and 120° ahead of V_p as shown by the vectors I_m, I_r, I_l in Fig.51b. In each position both dynamometers are read in watts, the ratio of the reading of D_p to that of D_s being **a**, **b**, and **c** respectively. Then,

$$a = \frac{V_p I_m}{V_s I_m \cos(\pi - \gamma)} = \frac{V_p}{-V_s \cos\gamma},$$

$$b = \frac{V_p I_r \cos\frac{2\pi}{3}}{V_s I_r \cos(\frac{5\pi}{3} - \gamma)} = \frac{-V_p \cos\frac{\pi}{3}}{V_s \cos(\frac{\pi}{3} + \gamma)},$$

$$c = \frac{V_p I_l \cos\frac{4\pi}{3}}{V_s I_l \cos(\frac{\pi}{3} - \gamma)} = \frac{-V_p \cos\frac{\pi}{3}}{V_s \cos(\frac{\pi}{3} - \gamma)}.$$

* A.Barbagelata, loc.cit., 1921.

remembering that γ is small

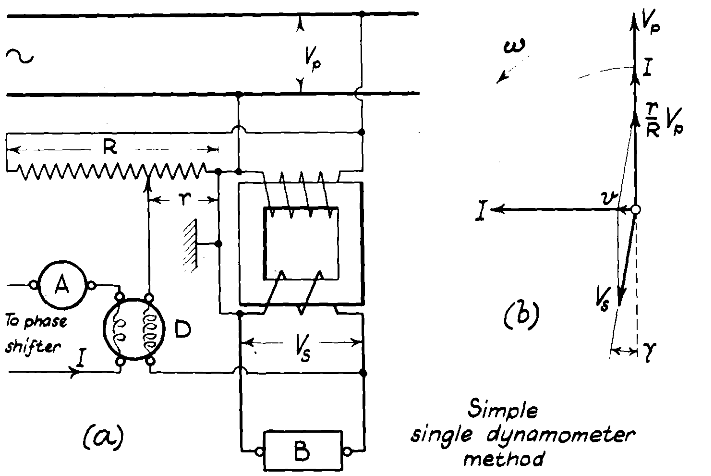
$$K_v \doteq a \qquad \text{numerically}$$

and $$\sin\gamma \doteq \frac{b-c}{2\sqrt{3}\,a} \doteq \gamma.$$

The method is subject to the same sources of error as the simple two dynamometer method, of which it is an adaptation suitable for use when a phase shifter is not available.

A further two dynamometer method is that used by Rosa and Lloyd* in one of the earliest precise investigations into the properties of voltage transformers. The two dynamometers are combined in a special instrument, a double dynamometer voltmeter, resembling in construction the two-element polyphase wattmeter. The instrument was of a reflecting pattern, two volt coils being mounted with planes at right angles upon a common suspension; each volt coil was provided with a fixed current coil for the excitation, the current coils also having their planes at right-angles. To measure the ratio, one volt coil is connected across the primary and the other across the secondary, each in series with its own current element and an appropriate resistance. By arranging the torques on the two elements to be in opposition zero deflection of the double voltmeter can be secured by adjustment of the resistances; the ratio is then easily obtained from the resistance settings. The phase-angle is found by using the two elements as a differential wattmeter. The apparatus gave results of high accuracy, but has not been adopted/

* E. B. Rosa and M. G. Lloyd, Bull. Bur. Stds., vol. 6, pp 1-30, 1910



R
r
A
To phase shifter
I
D
V_p
V_s
B
(a)
ω
V_p
I
$\frac{r}{R}V_p$
I
v
V_s
(b)
γ
Simple single dynamometer method

FIG. 52.

adopted by any other worker, so far as the writer is aware, presumably owing to the special character of the double reflecting dynamometer which is required.

3. Single dynamometer methods.

A single dynamometer can be readily employed to measure K_V and γ by making use of the potentiometer principle shown in Fig.52a.* The secondary voltage V_s is set in opposition to the fall of voltage in a portion r of a high resistance R connected across V_p. The voltage coil of the dynamometer is arranged to measure the resultant voltage. The auxiliary current I in the current coil of D is first set in phase with V_p and r is varied until D reads zero. Then, as Fig.52b shows, the resultant v of V_s and $\frac{r}{R}V_p$ is in quadrature with V_p and I. The phase of I is then advanced by 90° and the reading of D in watts is observed; if this be W then

$$W = vI = V_s I \sin\gamma$$

Further $$V_s \cos\gamma = \frac{r}{R} V_p$$.

whence $$K_V = \frac{R}{r}\cos\gamma \doteq \frac{R}{r}$$

$$\sin\gamma = \frac{W}{V_s I} \doteq \gamma$$

I is read on the ammeter A, while V_s can be taken from a voltmeter in the secondary burden B. I can be supplied from a phase shifter, as shown, or may alternatively be derived from the main and auxiliary phases of a two-phase source of supply.

In/

* L.T.Robinson, loc.cit.,1910 used the method for ratio measurements; C.H.Sharp and W.W.Crawford, loc.cit., 1911 for phase-angle determinations. The complete procedure is described by A.Barbagelata, loc.cit., 1921.

In a portable, self-contained apparatus designed by Allocchio and Bacchini[*] for testing voltage transformers on site the original adjustment of r is effected with the dynamometer excited directly from the supply. The quadrature reading is obtained by replacing the volt-coil series resistance by a large inductance of equal ohmic reactance. The need for a portable phase-shifter is thus avoided.

The main source of error arises from the fact that the voltage coil of D draws a definite current from the potential divider R. The coil must, in the first place, be non-reactive; this can be easily arranged by use of the ordinary condenser compensation. Craighead[†] has shown that the error in calculating the ratio from the above formula, i.e., neglecting the shunting influence of the volt-coil circuit of D on the voltage divider, can be neglected if the resistance of the detector circuit be about equal to $r(R-r)/R$. He further shows that if this value of volt-coil resistance be used, the actual phase-angle is about double the value found by calculation from the preceding formula. The method is, owing to this large correction, a poor one for angle measurements.

Palm[‡] avoids error due to this cause by the following process. The/

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* "Apparecchio per la misure del rapporto e dell' angolo di fase dei trasformatori," L'Elettro, vol.9, pp.344-346, 1922.

† J.R.Craighead, "Potential transformer testing. Note on the effect of the resistance of the detector circuit in determining the ratio of two alternating voltages, and the phase-angle between them, by the balance method," Trans.Amer.I.E.E., vol.31, pp. 1627-1633, 1912.

‡ A.Palm, Zeits.f.Inst., vol.34, pp.281-290, 1914.

The detector is applied directly to a portion r_s of R and the phase shifter adjusted until the reading of D is a maximum, d_1 say; then I and V_p are in phase. The connections of Fig.52a are then resumed and the reading of D brought to zero by adjustment of r . Applying the volt coil of D directly across r the phase-shifter is altered to make D read zero; then I and V_p are in quadrature. Finally the connections of Fig.52a are again resumed and a reading d_2 taken. Palm then shows that

$$K_V = \frac{V_p}{V_s} = \frac{R}{r}\cos\gamma \doteq \frac{R}{r},$$

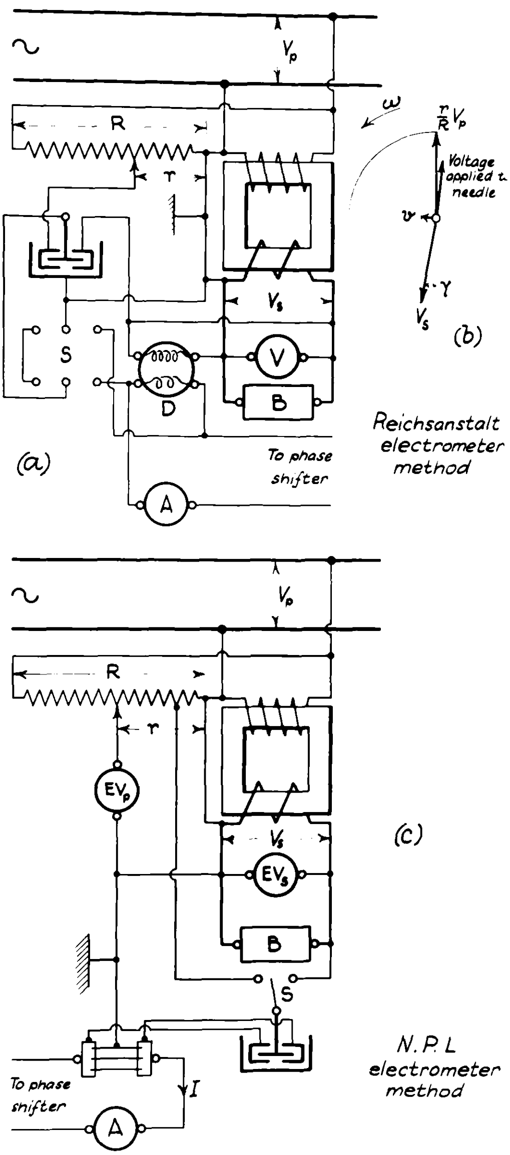
$$\tan\gamma = \frac{d_2}{d_1}\cdot\frac{R - r + \rho\frac{R}{r}}{R - r_s + w_s\frac{R}{r_s}},$$

where ρ is the resistance of the volt coil itself and w_s that of the whole volt-coil circuit.

4. Electrometer methods.

Both at the Reichsanstalt and at the National Physical Laboratory the quadrant electrometer has been developed for use in precision a.c. testing and takes the place of separately excited dynamometers in methods of testing voltage transformers. In the Reichsanstalt method* the needle and case of the electrometer are joined together and connected to the earthed common point of the windings of the transformer, Fig.53a. One pair of/

* E.Orlich, Elekt.Zeits., vol.30, pp.435-439, 466-470, 1909; H.Schultze, "Die Untersuchung von Spannungstransformatoren mittels des Quadrantelektrometers," Zeits.f.Inst., Vol.31, pp. 332-345, 1911.



(a)
R
r
V_p
V_s
S
D
V
B
A
To phase shifter
Reichsanstalt electrometer method
(b)
ω
$\frac{r}{R}V_p$
Voltage applied to needle
v
γ
V_s
(c)
EV_p
EV_s
I
N.P.L electrometer method

FIG. 53.

of quadrants is joined to the free secondary terminal; the other pair is connected to a tapping on a potential divider which has been put across the primary winding. The position of the tapping is altered until the electrometer reading is zero; then the ratio of the transformer is equal to the ratio of division of the potential divider by the tapping. The phase-angle is obtained by impressing a voltage of 50 to 100 volts between the needle and case from a phase shifter. The latter is regulated until the electrometer again reads zero; i.e., until the voltage between needle and case is in quadrature with v in Fig.53b. The angle is then read directly from the phase shifter or computed from the readings of the wattmeter D ammeter A and voltmeter V. With S to the left and the electrometer acting as a differential electrostatic voltmeter, at zero deflection since the needle is at zero potential

$$\left(\frac{r}{R}V_p\right)^2 - V_s^2 = 0$$

or

$$K_V = \frac{V_p}{V_s} = \frac{R}{r}$$

With S to the right and the electrometer reading zero as a differential electrostatic wattmeter when the phase shifter has been correctly adjusted the phase-angle is at once determined.

In the N.P.L. method* the voltage ratio is obtained from the readings of two electrostatic voltmeters, one connected across the secondary and one across a portion of the primary potential divider as shown in Fig.53c. These are reflecting instruments

* R.S.J.Spilsbury, Beama, vol.6, pp. 505-513, 1920.

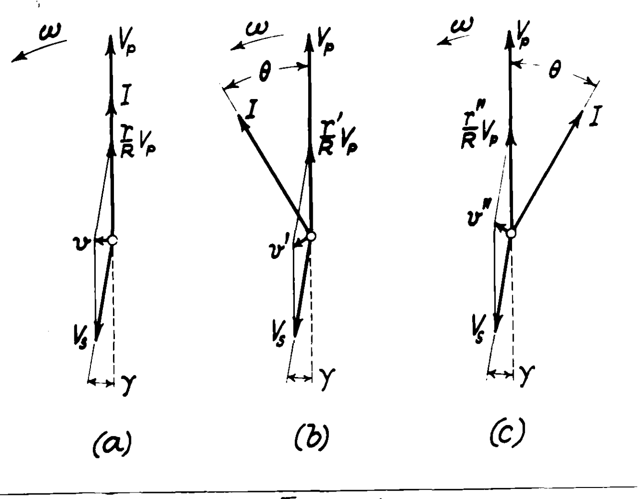
instruments reading up to 130 volts with scales divided to 0.01 volt, enabling the ratio to be found within 0.02%. The electrometer is used to determine the angle by throwing S to the left and adjusting the phase shifter until the instrument reads zero. If S be now thrown to the right the electrometer is being used as an electrostatic wattmeter reading W watts, where $W = V_s I \sin \gamma$, whence γ is found. As with all electrometer methods the possible accuracy is very high but the apparatus is of a very special character found only in these national laboratories.

CHAPTER III.

ABSOLUTE METHODS. NULL.

1. General.

All absolute null methods are based upon the potentiometer principle, in which the secondary voltage is set in opposition to the drop of voltage in a portion of a potential divider connected across the primary. The methods are divisible into two main classes. In the first the detector is a dynamometer, its volt coil connected to measure the resultant of the two voltages while its current coil is excited from a phase shifter. Since the secondary and primary voltages are not in direct opposition, but differ by the small angle γ, the resultant is never zero and there is always a small current passing through the detector coil; the reading of the dynamometer can, however, be always made zero by adjusting the phase shifter until the exciting current and the resultant voltage are in quadrature. Such methods, though based on the null principle, are not true balance methods; an example is given in Section 2. The second class contains true balance methods in which no current flows in the detector connection. This condition is secured by providing means of altering the phase of the current taken by the potential divider so that the voltage across the portion utilised is equal in magnitude and exactly opposite in phase to the secondary voltage. Suitable arrangements are considered in Section 3. The detector may be a separately/



ω
V_p
I
$\frac{r}{R}V_p$
v
V_s
γ
(a)
ω
V_p
θ
I
$\frac{r'}{R}V_p$
v'
V_s
γ
(b)
ω
V_p
θ
$\frac{r''}{R}V_p$
I
v''
V_s
γ
(c)

FIG. 54

separately excited dynamometer, synchronous commutator and d.c. galvanometer, or vibration galvanometer according to the practice of various experimenters; the last named instrument is by far the most convenient.

2. Single dynamometer method.

The method of Fig.52a is capable of null operation by adopting the following process, illustrated by the vector diagrams of Fig.54 a, b, and c. Firstly, with I in phase with V_p adjust r for null reading; then v and I are in quadrature. Now successively put I in advance and behind V_p by equal angles θ, in each case adjusting r to values r' and r'' to give null readings, showing that v' and v'' are normal to the appropriate I . These conditions are shown in Fig. 54 a, b, and c from the geometry of which

$$\frac{r}{R} V_p = V_s \cos\gamma,$$

$$\frac{r'}{R} V_p \cos\theta = V_s \cos(\theta+\gamma),$$

$$\frac{r''}{R} V_p \cos\theta = V_s \cos(\theta-\gamma).$$

From these it is easy to prove that

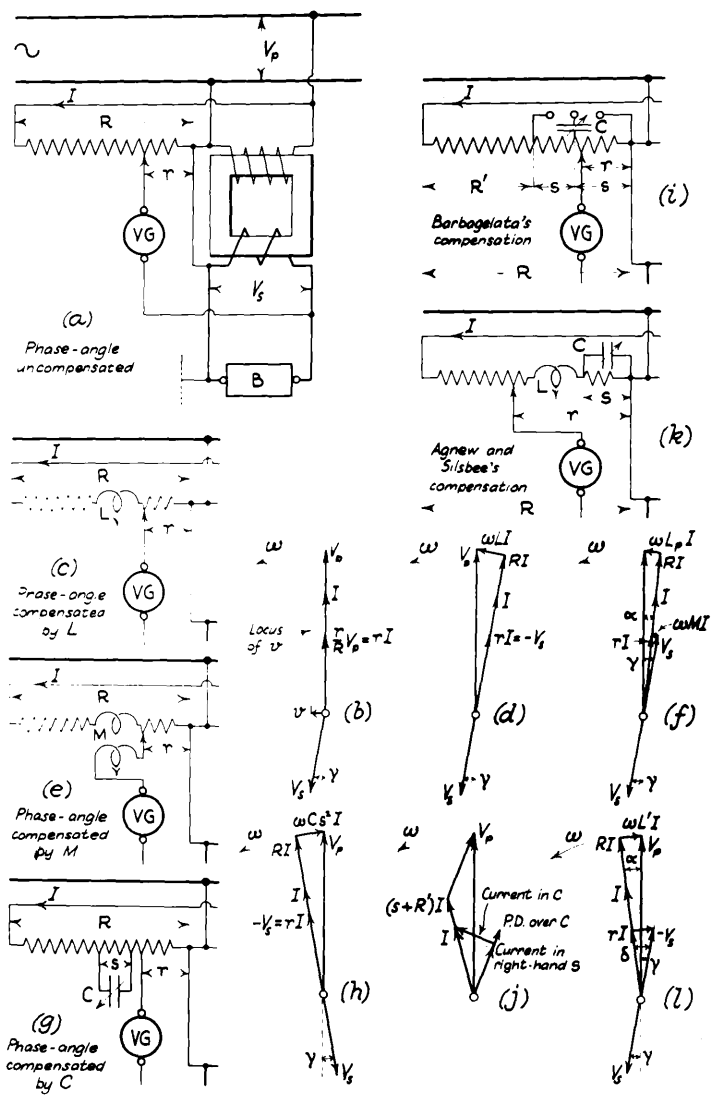
$$K_V = \frac{R}{r} \cos\gamma \doteqdot \frac{R}{r}$$

$$\tan\gamma = \frac{r''-r'}{2r\tan\theta}.$$

In particular if $\theta = 45°$.

$$\tan\gamma = \frac{r''-r'}{2r}$$

The method is not a true balance method since the detector or volt/



(a)
Phase-angle uncompensated
(c)
Phase-angle compensated by L
(e)
Phase-angle compensated by M
(g)
Phase-angle compensated by C
(i)
Barbagelata's compensation
(k)
Agnew and Silsbee's compensation
Locus of v
(b)
(d)
(f)
(h)
(j)
(l)
Current in C
P.D. over C
Current in right-hand S

Fig. 55.

volt coil carries a current in each case proportional to v, v' or v''.

3. True balance methods.

It is necessary now to examine true balance methods in which the detector, usually a vibration galvanometer, carries no current at the null indication. This necessitates the introduction of some auxiliary device to correct for the imperfect opposition of phase between V_p and V_s.[x]

3a. Phase-angle uncompensated. - If the phase angle between V_p and V_s is not allowed for, as in Fig.55a, it is only possible to get a minimum reading on the galvanometer if r be adjusted by moving the contact along the potential divider; this occurs when the resultant voltage v of $\frac{r}{R} V_p$ and V_s is normal to V_p, as Fig. 55b shows. Then $V_s \cos \gamma = \frac{r}{R} V_p$, from which $K_V = R/r$ very nearly. To get balance, i.e., a null indication, the current I in R must be displaced from V_p in the same direction as V_s is displaced from $-V_p$. This can be secured by including in R a suitable adjustable reactance which can take any of the forms described in the following subsections. By this means rI can be brought into direct opposition and into equality with V_s.

3b. Phase-angle compensated by L. - The most obvious way of producing the necessary shift in phase of I is to include in R a small variable self inductance, as shown in Fig.55c. Referring to Fig.55d it is obvious that for balance

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x Most of these devices are briefly suggested by C.H.Sharp and W.W.Crawford, loc.cit.ante., 1911.

$$rI = V_s ;$$

but $$I = \frac{V_p}{\sqrt{R^2+\omega^2L^2}} \doteq \frac{V_p}{R}$$

since ωL is generally small in comparison with R. Then

$$K_v \doteq R/r \quad \text{very nearly.}$$

Moreover, $$\tan\gamma = \frac{\omega L}{R} \doteq \gamma$$

It is apparent that this method of compensation only serves when γ is positive, i.e., when V_s lags on V_p reversed.

3c. Phase-angle compensated by M. - Compensation can be effected by putting a mutual inductance with its primary winding in series with the potential divider while its secondary is in the detector circuit, as shown in Fig.55e. In the vector diagram of Fig.55f, if L_p be the inductance of the mutual inductor primary, the current I will lag on V_p by a small angle, α, given by $\tan\alpha = \omega L_p/R$. The resistance drop rI and mutual e.m.f. ωMI must give a resultant equal in magnitude but opposite in phase to V_s. From the vectors,

$$V_s\cos(\gamma-\alpha) = rI$$

$$\tan(\gamma-\alpha) = \frac{\omega M}{R}.$$

But $$I = \frac{V_p}{\sqrt{R^2+\omega^2L_p^2}} \doteq \frac{V_p}{R}$$

and $$\cos(\gamma-\alpha) \doteq 1,$$

whence, $$K_v \doteq \frac{R}{r}$$

$$\tan(\gamma-\alpha) \doteq \gamma-\alpha = \frac{\omega M}{R}$$

Since/

Since M can be positive or negative the method enables either positive or negative, i.e., lagging or leading values of γ to be measured. The process is thus much more flexible and useful than that described in the preceding section.

3d. Phase-angle compensated by C. - If positive values of γ can be compensated by a self inductance it follows that negative angles can be allowed for by aid of a condenser, as is shown in Fig.55g. The impedance operator for the potential divider when a portion s is shunted by a condenser C is

$$z = R - s + \frac{s}{1+j\omega C s}$$
$$= R - s + \frac{s}{1+\omega^2 C^2 s^2} - j\,\frac{\omega C s^2}{1+\omega^2 C^2 s^2}$$

Now in practice $\omega^2 C^2 s^2$ is usually quite negligible in comparison with unity; hence

$$z \doteq R - j\omega C s^2$$

to a high degree of approximation. Hence the condenser has practically no effect on the resistance of the divider, but introduces a negative reactance which can be used to compensate for negative phase-angles. Thus

$$K_V \doteq \frac{R}{r}$$
$$\tan\gamma \doteq -\frac{\omega C s^2}{R} \doteq \gamma$$

The method suffers from limitation to cases when γ is negative, i.e., V_s leading on $-V_p$. Fig. 55h shows the approximate vector diagram.

The limitation can be removed by the device due to Barbagelata,* Fig.55i. The potential divider consists of

* A.Barbagelata, loc.cit., 1921.

a section R' in series with two equal resistances s, s, upon the right hand section of which the potential contact is applied. The condenser can be switched either across the left hand or the right hand member of the equal resistances. With C to the left the method is exactly like Fig.55g and serves to measure negative or leading angles. If C be put to the right then the vector relations of Fig.55j will hold. Here the common voltage across C and the right hand s is the vector difference between V_p and $(R'+s)I$, I necessarily leading on V_p. The current in C, leading 90° on this p.d. and the current in the right hand s must be equal to I; whence, as the vector diagram shows, the current in the right hand s, upon which the potential contact is set, lags on V_p and hence the arrangement will compensate positive or lagging phase-angles. Then as before

$$K_v \doteq \frac{R}{r}$$

and

$$\tan\gamma \doteq \frac{\omega C s^2}{R}.$$

taken as positive when the condenser is on the right and negative when on the left.

3e. <u>Phase-angle compensated by L and C</u> - Agnew and Silsbee* have used a compensation, shown in Fig.55k, which combines the methods of Fig.55 c and g; this enables both positive/

* P.G.Agnew and F.B.Silsbee, "The testing of instrument transformers," <u>Trans. Amer. I.E.E.</u>, vol. 31, pp. 1653-1638, 1912.

positive and negative values of γ to be dealt with. It will be observed that both L and C lie within the balancing section r of the potential divider. If $L' = L - Cs^2$ be the effective inductance of the section the approximate vector diagram will be as shown in Fig.551; from this

$$V_s = \sqrt{r^2 + \omega^2 L'^2} . I,$$

$$I = \frac{V_p}{\sqrt{R^2 + \omega^2 L'^2}};$$

neglecting $\omega L'$ in comparison with R and r,

$$K_v \doteqdot \frac{R}{r}$$

Again,

$$\gamma = \delta - \alpha = \arctan \frac{\omega L'}{r} - \arctan \frac{\omega L'}{R}$$

or since the angles are all small

$$\gamma \doteqdot \omega L' \left(\frac{1}{r} - \frac{1}{R} \right)$$

The method is the parallel in voltage transformer testing of the method of Schering and Alberti used in testing current transformers, see Fig.33a.

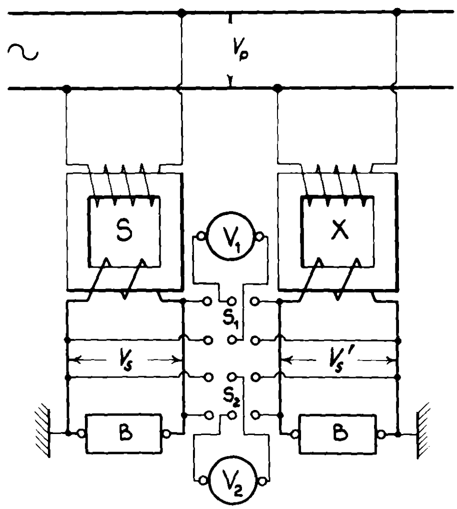
4. <u>A.C.Potentiometer method.</u>

Voltage transformers can be readily tested by the use of an alternating current potentiometer, when a suitable instrument is available. Since the range of such an instrument seldom exceeds 1.5 volts it is necessary to connect potential dividers across both primary and secondary windings and to select appropriate tappings upon them to get voltages falling within the range of the potentiometer. The process is then exactly the same as for current transformers, viz., the voltages across the/

the primary potential fraction and that across the secondary fraction are measured. From the known fractions of the potential dividers used the ratio is calculated at once. The phase-angle is obtained either by direct observation of the angle between the primary and secondary partial voltages or better still by measuring the angle between the resultant of these voltages and that measured on the primary. Let R_p be the resistance of the primary potential divider, r_p the resistance tapped off it, and v_p the measured voltage across r_p ; let the corresponding secondary magnitudes be R_s, r_s and v_s . Further let ϕ be the angle between v_p and the resultant of v_p and v_s ; this is much more easily measured than γ since it is a much larger angle. Then, if v be the magnitude of the resultant voltage,

$$K_v = \frac{v_p}{v_s} \cdot \frac{r_s}{r_p} \cdot \frac{R_p}{R_s}$$

$$\sin\gamma = \frac{v}{v_s} \sin\phi$$



Two voltmeter relative method

FIG. 56.

CHAPTER IV.

RELATIVE METHODS. DEFLECTIONAL.

1. Two voltmeter method.

Two voltage transformers of the same nominal ratio can be compared by the use of two voltmeters connected as in Fig.56. Here S is the standard transformer for which the ratio is known; X is the transformer which is to be compared with S. V_1 and V_2 are two similar voltmeters which can be put at will across the secondary of S or of X by operating the switches S_1 and S_2. With S_1 to the left and S_2 to the right let the secondary voltages of S and X be V_S and V_S' respectively; the reading of V_1 across the secondary of S will be v_1, and that of V_2 across X will be v_2'. Then if k_1 and k_2 be the correction factors of the voltmeter scales for these values of voltage

$$V_S = k_1 v_1 \quad \text{and} \quad V_S' = k_2 v_2'$$

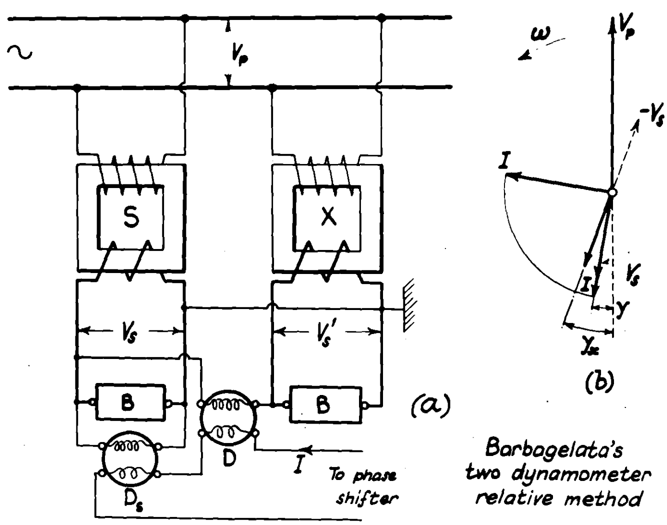
Now throw the switches over, thus interchanging the voltmeters with respect to S and X. Then since the instruments are similar V_S and V_S' will be practically unchanged but the readings will become v_2 and v_1' so that

$$V_S = k_2 v_2 \quad \text{and} \quad V_S' = k_1 v_1'$$

From these,

$$V_S^2 = k_1 k_2 v_1 v_2 , \qquad V_S'^2 = k_1 k_2 v_1' v_2' .$$

If/



Barbagelata's two dynamometer relative method

FIG. 57.

If V_p be the common primary voltage the ratio of S is

$$K_v = V_p/V_s$$

and of X
$$K_{vx} = V_p/V_s' = \frac{V_s}{V_s'} K_v$$

or
$$K_{vx} = \sqrt{\frac{v_1 v_2}{v_1' v_2'}} \cdot K_v$$

which eliminates the necessity for calibration of the voltmeters. The method is suitable for ratio tests on site but is not of very high precision unless the voltmeters are working high up on their scales. Its principal disadvantage is that it only determines the ratio of the unknown transformer.

2. Two dynamometer method.

Barbagelata* has given a simple two dynamometer method for comparing the ratio and phase-angle of one voltage transformer with that of another, see Fig.57a, analogous to that adapted for current transformers in Fig.39a. The current coils of the two dynamometers are excited in series from a phase shifter. The volt coil of D_s is joined across the secondary of the standard transformer S ; this dynamometer serves for checking the proper phase setting of the auxiliary current I and also to measure the secondary voltage V_s of S . The volt coil of the detector dynamometer D is joined to the secondary windings so that it measures the difference between V_s and V_s' . The current I is first set in phase with V_s , see Fig.57b, and the reading W_s of D_s observed. Then

* A.Barbagelata, loc.cit., 1921.

$$W_s = V_s I = \frac{V_p}{K_v} I \qquad \text{watts,}$$

from which $\quad V_p = \dfrac{K_v W_s}{I}$

At the same tíme the reading of D will be

$$W_1 = V_s I - V_s' I \cos(\gamma_x - \gamma)$$

The phase of I is now advanced by 90°, so that D_s reads zero; then the reading of D becomes

$$W_2 = -V_s' I \cos\left(\frac{\pi}{2} - \gamma_x + \gamma\right)$$
$$= -V_s' I \sin(\gamma_x - \gamma)$$

From W_1 , remembering that the angles are small

$$\frac{W_1}{I V_p} \doteqdot \frac{V_s}{V_p} - \frac{V_s'}{V_p}$$

whence,

$$\frac{1}{K_v} - \frac{1}{K_{vx}} \doteqdot \frac{W_1}{K_v W_s}$$

or $\quad K_{vx} \doteqdot \dfrac{K_v}{1 - \frac{W_1}{W_s}} \doteqdot K_v\left(1 + \frac{W_1}{W_s}\right)$

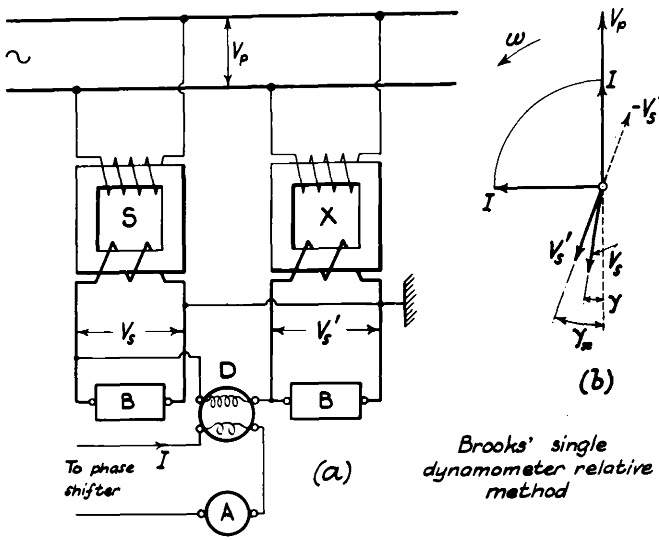
which determines K_{vx}. From W_2

$$-\frac{W_2}{V_s' I} = \sin(\gamma_x - \gamma) \doteqdot \gamma_x - \gamma$$

Now $V_s' = V_p / K_{cx}$,

$$\therefore \gamma_x - \gamma \doteqdot -\frac{W_2 K_{vx}}{V_p I} \doteqdot -\frac{W_2}{W_s} \cdot \frac{K_{vx}}{K_v}$$

which determines the difference between the phase-angles of the transformers.



Brooks' single dynamometer relative method

FIG. 58

3. Single dynamometer method.

Brooks* has described a single dynamometer method for comparing voltage transformers, shown in Fig.58a, which proves very useful for tests to be made on site. The dynamometer D may conveniently be a low reading precision wattmeter. Readings of D are taken first with the current coil of the instrument carrying a current in phase with V_p and secondly when this current is advanced in phase by 90° as shown in Fig.58b. The voltage coil is excited by the two secondaries in opposition. Then if W_1 and W_2 be the two readings of D in watts,

$$\begin{aligned} W_1 &= V_s I \cos(\pi-\gamma) - V_s' I \cos(\pi-\gamma_x) \\ &= I\left[V_s'\cos\gamma_x - V_s\cos\gamma\right]; \end{aligned}$$

$$\begin{aligned} W_2 &= V_s I \cos(\tfrac{\pi}{2}-\gamma) - V_s' I \cos(\tfrac{\pi}{2}-\gamma_x) \\ &= I\left[V_s\sin\gamma - V_s'\sin\gamma_x\right]. \end{aligned}$$

Dividing the first by V_s,

$$\frac{W_1}{V_s I} \doteq \frac{V_s'}{V_s} - 1 \doteq \frac{K_v}{K_{vx}} - 1,$$

so that the ratio is

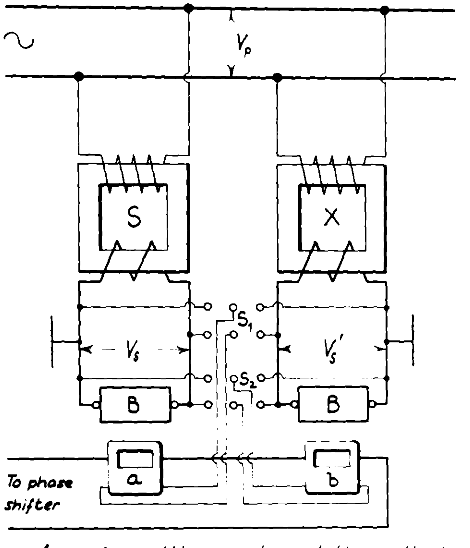
$$K_{vx} \doteq \frac{K_v}{1+\frac{W_1}{V_s I}} \doteq K_v\left(1-\frac{W_1}{V_s I}\right)$$

Similarly dividing the second, and remembering that V_s' and V_s are nearly the same

$$\gamma_x - \gamma \doteq -\frac{W_2}{V_s I}$$

V_s is taken from the reading of a voltmeter in the burden of the/

* H.B.Brooks, "Testing potential transformers," Bull.Bur.Stds., vol.10, pp. 419-424, 1914; F.A.Kartak, loc.cit.ante.,1920; H.M.Crothers, Elec.World, pp. 119-121, 1919.



Agnew's watthour meter relative method

FIG. 59.

transformer S ; I is read on the ammeter A.

In order to determine which transformer has the smaller ratio and which the smaller phase-angle, Brooks suggests the following procedure. With no load on the secondary of X arrange the reading to be up the scale when making the observation of W_1 . If on applying load the reading increases then the ratio of X at no load is less than that of S . When taking the value of W_2 , if X be further loaded its secondary current will be caused to lag more relative to V_p . If, therefore, the reading is increased on adding non-inductive load to X the secondary voltage thereof lags behind that of S , i.e., γ_x exceeds γ .

The method gives results in good agreement with laboratory methods, is quick, and has the great practical advantage of using only portable instruments of ordinary commercial pattern.

4. Agnew's watt-hour meter method.

Agnew's method, using two matt-hour meters, described in Section 4 of Chapter V, Part II, is immediately applicable to tests on voltage transformers. Referring to Fig.59 the similar meters a, b are arranged so that their voltage circuits may be connected at will to the secondary of either transformer by operation of the switches S_1, S_2. The current circuits are supplied in series from a phase shifter. The process is exactly the same as that used with current transformers. With the phase shifter adjusted so that the meters work at unity power-factor the revolutions made in a given time by the meter discs are observed/

observed (i) with meter a joined to S and meter b to X ; (ii) with a joined to X and b to S . Then, if a_s, b_x, a_x, b_s be the observed numbers of revolutions, the analysis given in Part II at the Section cited shows that

$$\frac{K_{vx}}{K_v} \doteqdot \sqrt{\frac{a_s}{a_x} \cdot \frac{b_s}{b_x}},$$

eliminating the rates of the meters. A second set of readings, a'_s, b'_x, a'_x, b'_s is then taken when the phase shifter is

less; if ϕ be the phase-displacement then as before or of 0.5 or

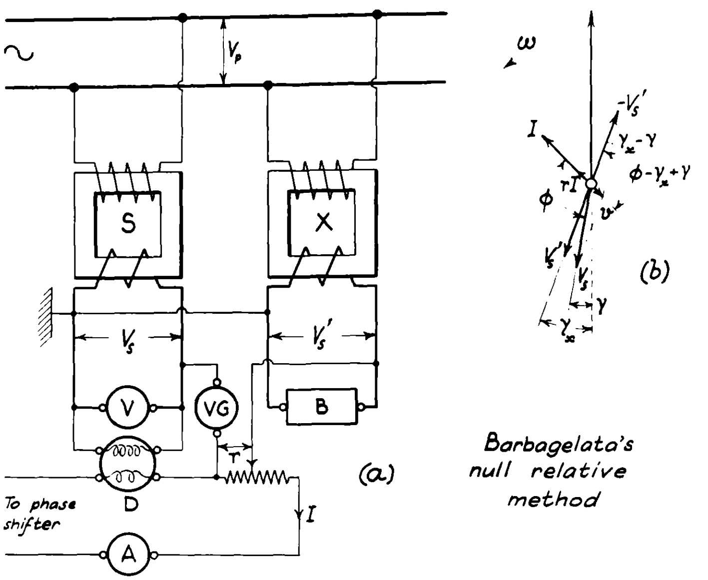
$$\gamma_x - \gamma \doteqdot -\frac{1}{2\tan\phi}\left[1 - \frac{a'_x b'_x}{a'_s b'_s} \cdot \frac{a_s b_s}{a_x b_x}\right].$$

ore

$$\gamma_x - \gamma \doteqdot -\frac{1}{2\tan\phi}\left[1 - \frac{\quad}{a'_s b'_s} \cdot \frac{\quad}{a_x b_x}\right].$$

The signs are correct if both secondary voltages lag on $-V_p$ and $\gamma_x > \gamma$. To test which transformer has the greater errors, add a further non-inductive load to X ; then if the differences are increased X had originally the greater errors, since the addition of load increases the ratio and phase-angle of a transformer.

As in the case of current transformers high precision is attainable, but the method is rather slow. It is, however, invaluable for tests on site where high accuracy is requisite.



Vp
S
X
Vs
Vs'
V
VG
B
D
A
r
I
To phase shifter
(a)
ω
-Vs'
γx-γ
φ-γx+γ
rI
φ
v
γ
γx
(b)
Barbagelata's null relative method

FIG. 60.

CHAPTER V.

RELATIVE METHODS. NULL.

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1. Barbagelata's Method.

Null relative methods of comparing two voltage transformers are not numerous, although it would appear to be possible to compare the secondary voltages of two transformers by any of the null processes described in Chapter III for balancing the primary and secondary voltages of a single transformer. Indeed, Agnew and Fitch in their classic paper on instrument transformer testing suggest such a differential method; they point out, moreover, that since the test is like that of comparing the primary and secondary voltages of a 1/1 transformer, the resistances in the detector ciruit are much smaller than in the absolute method. Consequently, with the same apparatus, the sensitivity goes up enormously. Nevertheless, the only null relative method definitely described in the literature of the subject is that due to Barbagelata[x] and illustrated in Fig.60a. The dynamometer is used to check the phase setting of the phase shifter; regulation of the latter and of the value of r enables balance, *i.e.*, null indication on the vibration galvanometer, to be secured. Let W be the reading of D in watts, and V_s the reading of V, then $W = V_s I \cos\phi$, where ϕ is the phase difference between V_s and I at balance. From the geometry of the vector diagram of Fig.60b, since/

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x A.Barbagelata, loc.cit., 1921.

since rI balances the resultant v of V_s and $-V_s'$,

$$rI\cos(\phi-\gamma_x+\gamma)+V_s\cos(\gamma_x-\gamma)=V_s',$$
$$rI\sin(\phi-\gamma_x+\gamma)=V_s\sin(\gamma_x-\gamma).$$

Neglecting $\gamma_x-\gamma$ in comparison with ϕ and remembering throughout that the angles γ_x and γ are very small, the first relation becomes

$$rI\cos\phi+V_s \doteqdot V_s'$$
$$\text{or}\qquad V_s'-V_s \doteqdot rI\cos\phi$$

Dividing by $V_p=K_vV_s$ and substituting for $I\cos\phi$ from above

$$\frac{1}{K_{vx}}-\frac{1}{K_v}\doteqdot\frac{rW}{K_vV_s^2},$$

whence

$$K_{vx}\doteqdot K_v\Big/\left(1+\frac{rW}{V_s^2}\right)\doteqdot K_v\left(1-\frac{rW}{V_s^2}\right).$$

For the angle error, the second relation gives,

$$rI\sin\phi.\cos(\gamma_x-\gamma)\doteqdot V_s\sin(\gamma_x-\gamma),$$
$$\tan(\gamma_x-\gamma)\doteqdot\frac{rI}{V_s}\sin\phi$$

and finally,
$$\gamma_x-\gamma\doteqdot\frac{r}{V_s^2}\sqrt{V_s^2I^2-W^2}$$

2. <u>A.C.Potentiometer method</u>.

If an alternating current potentiometer is available the ratios and phase-angles of two voltage transformers can be compared by measuring the magnitude and phase of the secondary voltages of the two transformers and also that of the vector difference of these voltages in the usual way.

PART IV.

CHOICE OF METHODS FOR TESTING INSTRUMENT TRANSFORMERS.

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CHAPTER I.

CURRENT TRANSFORMERS.

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1. Preliminary.

In Part II a full review and classification of all the numerous methods available for current transformer testing has been made. This will be of value to the student or research worker who, in making a study of any subject in which current transformers are involved, desires to find out quickly and briefly the principle underlying any method to which he may find reference in technical literature. That is to say, Part II is primarily sufficient in researches where an historical point of view is involved. The matter is quite different, however, when the subject is approached from a practical standpoint. Many of the methods described in the text are of purely historical interest; others, though good in themselves, have been superseded by methods of greater practical convenience and value. Consequently, the question that interests the testing engineer, and all other workers who require to make measurements of instrument transformer characteristics, is to determine what methods are suitable for making such measurements in various practical circumstances. It is the object of the present Chapter to show how to choose methods for current transformer testing which meet the requirements of modern practice; a similar purpose is fulfilled by Chapter II of this Part with regard/

regard to voltage transformers.

The methods required in testing are of two main classes, (i) High precision or laboratory methods and (ii) Methods for tests on site. In each class absolute and relative methods are used. The first class makes use of all the resources of a laboratory or test-room, sensitive detectors, refined procedure, and all auxiliary aids to high precision coupled, especially in the works test-room, with speed and convenience in operation. The second class makes use of apparatus which must be portable, and as the observations are made by the aid of pivoted pointer-instruments the order of precision is necessarily lower than that of the first class.

2. Methods for laboratory tests.

For tests of the highest order of precision two methods are pre-eminent, Agnew and Fitch's method of Fig.16b and the Electrometer methods of Fig. 23. These methods are of the absolute type and are capable of extreme refinement. They have been used in some of the most thorough and accurate investigations into the properties of current transformers and are well suited for research work of a very precise character. The first method has been adopted at the Bureau of Standards; forms of the electrometer method occupy a similar place in the Reichsanstalt and the National Physical Laboratory. For ordinary laboratory work, and still more so in the works test room such methods are too precise and too troublesome; the first method utilises a delicate reflecting dynamometer, the second method an electrometer of very special construction/

construction, both of which require a too elaborate technique for their satisfactory use in any but the most favourable conditions, such as would be found in a great national laboratory.

Of exact laboratory or test-room methods of the absolute type undoubtedly the most suitable is that due to Schering and Alberti, see Fig.33a, developed for high accuracy routine tests at the Reichsanstalt. This method uses very simple apparatus, easily procured in any test-room, and is almost entirely free from residual or inductive interference errors. It is very easy to use and very quick. Next in order of suitability is the method due to Sharp, Fig.32a, which has been developed by the Bureau of Standards for routine work of high precision. The adjustments are again simple and tests can be quickly made; residual errors are slight, but the method is somewhat susceptible to stray field interference. This can be largely avoided by use of an astatic mutual inductor and by careful setting up of the test circuit; full details are given in the text. A third method, adopted by the Westinghouse Co. of America, is that due to Sharp but perfected by Fortescue and shown in Fig.35a. This method has great technical advantages but has one important practical disadvantage, namely, its cost. It is essential that the mutual inductances be made <u>perfectly</u> astatic; this is only ~~responsible~~ possible by building them of toroidal design, making them very expensive to construct. It will be noted that all these absolute methods are of the null type, requiring for their use a vibration galvanometer. Moreover, all have the advantage that/

that no auxiliary source or phase shifting device is necessary.

In some cases, e.g., in standardising a large number of transformers for use with precision ammeters or wattmeters, a relative method may be preferred; the unknown transformers can by such a method be rapidly compared, one after the other, with a standard transformer, the ratio and phase errors of which have been accurately determined by an absolute method. Such relative methods have an important advantage in routine works testing, namely, that a more robust and less sensitive detecting instrument is required than is the case in absolute methods. This is obvious when it is remembered that the detector is now required to measure the difference between the characteristics of the two transformers and not the absolute value of those characteristics. Consequently the sensitivity required falls within that which can be obtained from good pivoted instruments; the detector usually used is a low reading wattmeter. Of these methods the deflectional method of Silsbee, Fig.40, is much used and Spilsbury has recently devised a portable detector specially for works use; the method is, of course, deflectional. Where a null relative method is desired, using either a vibration galvanometer or a wattmeter excited from a two-phase auxiliary supply as detector, Silsbee's null modification of the method just mentioned is very suitable, see Fig.43. When it is desired to use a single standard transformer to test a variety of transformers of ratios differing widely therefrom Barbagelata's method/

method of Fig.44 is very useful.

If an a.c. potentiometer be available in the laboratory this instrument can be very conveniently used to measure absolutely the ratio and phase-angle of a transformer, as in Fig.36, or to compare the characteristics of one transformer with those of another by the process shown in Fig.45.

3. Methods for tests on site.

The essential qualities demanded of a method for testing current transformers on site are (i) adequate accuracy, (ii) small amount of apparatus, (iii) portability and robust construction of apparatus, and frequently (iv) non-interruption of the supply while the test is being made. This last requirement can usually be met by cutting the transformer out of service by means of a jumper connected across its primary side, making the requisite test connections and then removing the jumper so that the transformer is excited by and tested at its load current. The methods, also, should be as far as possible independent of fluctuations in the current.

It is usual to prefer relative methods for tests to be made on site, the one best meeting the desired conditions being Agnew's watthour meter method of Fig.42. For this are required the standard transformer, two calibrated watt-hour meters, two throw-over switches, phase-shifting transformer, and a watch. Results can be obtained with a high degree of accuracy, the main disadvantage of the method being its slowness, about 6 to 7 hours being required for a normal test. Greater speed is attained/

attained by non-integrating methods such as that of Silsbee shown in Fig.40a. The apparatus required for this consists of ammeter, voltmeter and wattmeter for phase settings, standard transformer, phase-shifting transformer, and detector dynamometer. Crother's modification of this method, Fig.41a, possesses the advantage that the phase-shifter is replaced by a condenser, the wattmeter by a low resistance, while the voltmeter is unnecessary. The three-phase methods of Barbagelata, Fig.17a, and Harned, Fig.18a, have the great practical advantage of using for excitation of the detector dynamometer the available three-phase supply. The former method is one of the simplest, since all that is required is a suitable pair of dynamometers and two three-way switches.

4. Practical Precautions.

Whether made in the test-room or on site, tests must be carried out with regard to certain points of practical procedure if accuracy of the results is to be guaranteed. It is almost invariably the case that the available source of supply is of too high a voltage for direct application to the transformer under test; hence it is usual to interpose between the source and the test circuit a suitable step-down transformer. This may be very conveniently an inverted current transformer similar to that being tested. Unless the object of the test is to locate some defect in magnetic characteristics, the transformer under test should be carefully demagnetised, as described in Section 5c of Chapter I, Part II, before the observations of ratio and phase/

phase-angle are made. Again, it is usually necessary to see that the relative polarities of the primary and secondary windings are correct; this is automatically checked in those methods employing dynamometer instruments. In other cases this matter must be made the subject of a special test with the aid of suitable wattmeters; in no case should a direct current be used for the purpose, as is often recommended, unless the core be thoroughly demagnetised after the polarity check has been made. Before the test proper is commenced the whole set-up must be examined for inductive interference effects; such adjustments in the lay-out of the circuit must be made to reduce these to the minimum possible. This precaution is particularly essential when transformers for large primary current are being tested and in those methods where self or mutual inductors form an essential feature, unless it is certain that such inductors are perfectly astatic. In general, it may be said that the utmost care must be exercised in assembling the test circuit and arranging the apparatus.

Before testing, the secondary burden must be adjusted to its proper value. A complete specification of the inductance and resistance of the burden should be made; a mere statement of the volt-amperes absorbed by it is quite insufficient. The frequency of the supply must be measured, maintained constant, and stated as a necessary condition of the test. It is usual to take observations of ratio and phase-angle at secondary currents/

currents of 0.5, 1, 2, 3, 4 and 5 amperes; experience shows that the characteristics vary with sufficient regularity to enable smooth curves to be drawn through the results for these points from which the values at intermediate currents can be interpolated.

CHAPTER II.

VOLTAGE TRANSFORMERS.

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1. Preliminary.

Methods for testing voltage transformers are much less numerous and of considerably greater simplicity than those introduced for testing current transformers. Consequently the task of selecting the methods best adapted to the uses of laboratory practice and of testing on site is an easy one, and will be undertaken in the following sections.

2. Methods for laboratory tests.

For absolute measurements of the highest precision the dynamometer method of Agnew and Fitch, Fig.50, or the electrometer method, Fig.53 are available. These, however, necessitate the use of very special apparatus, reflecting dynamometers in the one case and precision electrometers in the other. Absolute null methods, using such a detector as a vibration galvanometer, do not require any very specialised material and enable measurements to be made with almost equal precison. The best of these null methods are (i) that with mutual inductance compensation for the phase-angle, Fig.55e, and (ii) the method of Agnew and Silsbee, Fig.55k, in which the compensation is effected with the aid of a condenser and self inductance. A further useful null method is that of Barbagelata, Fig.55i.

Some such absolute method must be employed when it is desired/

desired to determine, <u>ab initio</u>, the characteristics of a transformer intended to be utilised as a standard. Once a calibrated standard transformer is available, a relative method may be used with even greater advantage than is the case in current transformer testing. Firstly, in relative methods measurements are made exclusively upon the low voltage or secondary sides of the transformers, no connection to the high voltage primary side being required. Consequently much greater safety for the operator is secured. Moreover, since all high resistance potential dividers are eliminated and connections are made directly to the secondary circuits the sensitivity with given instruments is very much higher than if these instruments were used in an absolute test; so great is the increase that it becomes possible to attain quite sufficient precision by substituting good pointer instruments for the reflecting type of instruments essential in absolute methods.

Of laboratory relative methods the two dynamometer method of Fig.57 and the single dynamometer method of Fig.58 are available; both are deflectional. Barbagelata's method of Fig.60 is a satisfactory null process.

3. Methods for tests on site.

For testing transformers on site portability of the apparatus and adequate sensitivity with pointer instruments are essential features. The last named condition renders relative/

relative methods of special value, still more so since in such methods greater safety is obtained than is possible in absolute methods because of the fact that no part of the measuring apparatus is connected to the high voltage side. Consequently relative methods are generally preferred. Of these the Brooks single dynamometer method of Fig.58 serves for most tests and is quick to use. Agnew's watt-hour meter method of Fig.59 is applicable to tests of high accuracy but is rather slow.
~~rather slow.~~

4. Practical precautions.

Just as in current transformer testing,it is necessary to assemble the test circuit so that residual errors and the effects of inductive interference are reduced to the least possible amount. Further, since the primary side of a voltage transformer is supplied at a high voltage the circuit must be arranged to give the greatest safety for the observer. In absolute methods it is essential that the common point of the primary and secondary winding be earthed; the balancing adjustments must be put into the test circuit in such a position as to be as near earth potential as possible. All high voltage portions of the circuit must be put well out of reach. Relative methods have the advantage that the primary side of the transformers contains no part of the measuring gear; nevertheless one point in the test circuit connected to the secondaries must be earthed in order to guard against the potential of the entire secondary circuit being raised above earth potential by electrostatic induction from the primary.

In/

In carrying out tests it is essential to state the voltage, frequency, and the amount and nature of the secondary burden for which the ratio and phase angle figures are found.

ADDITIONAL PAPERS

IN SUPPORT OF THESIS FOR THE DEGREE OF Ph.D

by B.Hague, M.Sc., D.I.C.

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1. "The application of the roots of unity in the theory of balanced and unbalanced polyphase circuits". This paper has recently been accepted by the Institution of Civil Engineers for publication, in a slightly modified form, as a Selected Engineering Paper.
2. "The dynamometer wattmeter. Some notes on its theory-Application of corrections". Reprinted from __Electrician__, Jan., 1924.
3. "Measurement of dielectric losses at high voltages. A new type of alternating current bridge". Reprinted from __World Power__, Aug., 1925.
4. "Alternating current bridge methods for the measurement of inductance, capacitance, and effective resistance at low and telephonic frequencies", pp.i-xiii, 1-302, Sir Isaac Pitman, 1923. See leaflet, and separate copy of the book.

The above papers have a direct bearing on the subject of the thesis. Other papers, dealing with different topics, will be submitted to the Examiners if required.

THE APPLICATION OF THE ROOTS OF ~~UNITY~~ IN THE THEORY OF BALANCED AND UNBALANCED POLYPHASE CIRCUITS.

by

B. Hague, M.Sc., D.I.C., A.M.I.E.E.

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— CONTENTS. —

cal coordinates.

Scalar relations.

Unsymmetrical two-phase systems in quarter-phase coordinates.

Scalar relations.

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Negative sequence.

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11. TECHNICAL APPLICATIONS OF SYMMETRICAL COORDINATES.

The practical designation of unbalance.

The measurement of negative sequence current.

The measurement of power in unsymmetrical systems.

Derivation of single-phase supply from a polyphase network.

The air-gap field of a three-phase winding.

12. CONCLUSION.

SUMMARY.

The object of the paper is to explain the elementary principles of certain powerful methods of analysis applicable to the theory of symmetrical and unsymmetrical polyphase systems, these methods being,as yet,but little used by British engineers.

Following a brief introduction, Sections 2 and 3 contain the elementary principles of the vector theory of alternating currents. These principles are applied in Section 4 to the treatment of symmetrical polyphase systems,the series of complex operators,equivalent to the roots of unity,which form the basis of the method being defined and tabulated. The application of these operators to the representation of harmonics in polyphase circuits is made in Section 5,while the influence of star and mesh interlinkage on the currents and voltage harmonics is investigated in Section 6.

b An explanation of Fortescue's method of resolving unsymmetrical polyphase systems into a number of superposed symmetrical systems is given in Section 7,with an application to the detailed treatment of the three-phase circuit in Section 8. The resolution of hemisymmetrical systems,notably in the balanced and unbalanced quarter-phase circuit,into the sum of symmetrical systems is made in Section 9.

The paper concludes with a short treatment in Section 10 of the way in which power is transmitted in unsymmetrical polyphase circuits,followed by a brief statement in Section 11 of a number of technical problems which are most advantageously attacked by the methods explained in previous sections.

Note Concerning Notation.

In reading the mathematical work symbols underlined in red, thus, $\underline{e}$, $\underline{i}$, $\underline{r}$, are to be taken as representing Clarendon type.

Symbols simply underlined in black, thus, $\underline{E}$, $\underline{I}$, $\underline{r}$, are to be taken as representing Italic type.

Greek letters are inserted by hand, thus, λ, ϵ ; as also are the symbols in indices, such as $\lambda^{(N-1)}$, $\epsilon^{j\frac{2\pi}{N}}$. In the latter the numbers are supposed to be in ordinary type and the letters in Italic.

THE APPLICATION OF THE ROOTS OF UNITY IN THE THEORY OF BALANCED AND UNBALANCED POLYPHASE CIRCUITS.

by B.Hague, M.Sc., D.I.C., A.M.I.E.E.

Lecturer in Electrical Engineering in the University of Glasgow.

1. INTRODUCTION

The recent appearance of an interesting article by Professor H.J.S. Heather*, in which the theory of d.c. armature windings is approached from an original point of view, draws attention to a principle of considerable utility in polyphase theory which does not seem to have received from students and engineers the attention that ot deserves. Briefly, the principle involved is the representation of a number of points spaced uniformly round the circumference of a circle by the use of an artifice, well known to mathematicians, based on the geometric properties of certain sequences of complex numbers. The principle was early adapted to the representation of polyphase systems of vectors by the late C.P.Steinmetz*, and has received some remarkable generalisations, chiefly in the work

*H.J.S.Heather, "A contribution to the theoby of direct current armature winding", *World Power*, vol.1, pp.158-163, March, 1924.

*C.P.Steinmetz, *Theory and calculation of alternating current phenomena*, 3rd. edition, pp.434-436 and p.495, 1900 of Charles L.Fortescue* and other American writers. This work

*C.L.Fortescue,"Method of symmetrical coordinates applied to the solution of polyphase networks",Proc.Amer.I.E.E.,vol.37, pp.629-716,1918.

does not seem to be fully appreciated in this country,in spite of the simplicity which it introduces into the theory of unbalanced polyphase systems.

It is the object of the author in this paper briefly to review the use of the roots of unity in the treatment of polyphase systems,to explain the application of Fortescue's method of symmetrical coordinates to unsymmetrical systems,and to discuss certain important technical uses of the method. It is hoped that by these means engineers may be interested in the utility of these methods of approaching polyphase theory. To fix ideas the subject is developed with reference to alternating currents and voltages,using the vector notation,but the discussion applies with equal force to any other physical quantity,whether electrical or otherwise,that can be treated by a harmonic vector algebra.

2. VECTOR NOTATION.

A vector quantity,having magnitude,direction and sense can be graphically represented by a vector $\mathbf{r}$ of length r inclined at an angle θ to the axis of X. The Clarendon letter throughout denotes the entire vector and symbolises its magnitude,direction and sense; the Italic letter symbolises the scalar magnitude only of the vector. If the components of $\mathbf{r}$ be be $\underline{a}$ and $\underline{b}$ parallel to the axes of X and Y respectively then,

$$\underline{r}=\underline{a}+\underline{b}$$

is the vector equation for $\underline{r}$ in terms of its rectangular components, as illustrated in Fig.1. If now $\underline{x}$ is a vector of unit length measured along OX and the symbol $\underline{j}$ is used to denote the operation of rotating a vector through a counterclockwise angle of $\pi/2$ radian without changing its magnitude, then $\underline{jx}$ will denote a vector of unit length measured along OY. Then if a be the number of times the unit vector $\underline{x}$ is contained in the vector $\underline{a}$, and b is the number of times $\underline{jx}$ is contained in $\underline{b}$, then $\underline{a} = a\underline{x}$ and $\underline{b} = \underline{j}b\underline{x}$, so that

$$\underline{r} = (a+\underline{j}b)\underline{x}. \qquad (1.$$

Now from the definition of $\underline{j}$, and as shown by Fig.1, two operations with $\underline{j}$ will rotate a vector through π radians, i.e., $\underline{jjx}=-\underline{x}$; further $\underline{jjjx} = -\underline{jx}$, and so on. Hence denoting successive operations with $\underline{j}$ by powers of $\underline{j}$,

$$\underline{jx} = \underline{jx},$$
$$\underline{j}^2\underline{x} = -\underline{x},$$
$$\underline{j}^3\underline{x} = -\underline{jx},$$

and so on. Thus the relation between powers of $\underline{j}$ and $\underline{j}$ is the same as that connecting the corresponding powers of $\sqrt{-1}$ with $\sqrt{-1}$. Note that this is not equivalent to saying that $\underline{j}$ and $\sqrt{-1}$ are identical; the former denotes an operation, while the latter is an imaginary numeric. The correspondence, however, makes it possible to treat the effect of $\underline{j}$ in arithmetical expressions as equivalent to multiplication by $\sqrt{-1}$.

The complex operation denoted by $(a+jb)$ applied to a unit vector x multiplies its magnitude by $r = (a^2+b^2)^{\frac{1}{2}}$ and rotates it through a counterclockwise angle $\theta = \tan^{-1} b/a$. Now $a = r\cos\theta$ and $b = r.\sin\theta$, so that $(a+jb) = r(\cos\theta + j.\sin\theta)$. Remembering the arithmetic property of the operator j and applying De Moivre's theorem gives

$$r = (a+jb)x = r.\epsilon^{j\theta} x, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2.$$

as equivalent symbols for the complex operation on the unit vector. The tensor element r alters the length and the rotor element $\epsilon^{j\theta}$ the orientation of the given vector operand.*

---

*~~It should be noted that this exponential notation for the rotor element of a complex operation is symbolic only, and the symbol ε must not be given the customary arithmetical significance. It must be thought of as representing the exponential function expψ in the form εψ, when ψ is any number, real or complex. See G.H. Hardy, A course of pure mathematics, 2nd edition, Chapter IX, X, 1914.~~ To verify the transformation just made, readers not familiar with De Moivre's theorem may proceed thus. It is proved in mathematical books that

$$\exp\psi = \epsilon^{\psi} = 1+\psi+\frac{\psi^2}{2!}+\frac{\psi^3}{3!}+\frac{\psi^4}{4!}+ \ldots\ldots\ldots\ldots .$$

Insert $\psi = j\theta$

$$\epsilon^{j\theta} = (1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}- \ldots.) + j(\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}- \ldots.)$$
$$= \cos\theta + j.\sin\theta,$$

from the well-known expansions of $\cos\theta$ and $\sin\theta$ for real angles.

These operations are not restricted to a unit operand. Suppose a vector r of length r and given position is multiplied in length r'/r times and advanced through an angle ϕ as shown in Fig.2, thereby becoming the vector r'. Then the relation between the two vectors is

$$r' = \frac{r'}{r}.\epsilon^{j\phi} r, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3.$$

a theorem of considerable utility in later work.

3. ALTERNATING CURRENTS IN VECTOR NOTATION.

These general vector principles can now be applied to the representation of sinusoidally varying currents. If $\underline{i}_1$ be the maximum value of an harmonically varying current of which the instantaneous value is $\underline{i} = \underline{i}_1 \cos \omega \underline{t}$, ω being the pulsatance,* the current can be represented in the following way. Let a vector $\underline{i}$ of length $\underline{i}_1$, starting initially along OX in Fig.3, rotate with angular velocity ω about O; then the component of $\underline{i}$ along OX at any instant has a magnitude $\underline{i}_1 \cos \omega \underline{t}$, equal to the instantaneous value of the current. Such a vector is called an 'harmonic vector' and in terms of the unit vector $\underline{x}$ is given by the equation

$$\underline{i} = \underline{i}_1 (\cos \omega \underline{t} + \underline{j}.\sin \omega \underline{t}) x = \underline{i}_1 \epsilon^{j\omega t} \underline{x},$$

by simple changes in Equation 2. A precisely similar notation applies to the representation of alternating voltages.

*The pulsatance of an alternating quantity is 2π times its frequency.

3. SYMMETRICAL POLYPHASE SYSTEMS OF CURRENTS IN VECTOR NOTATION.

A system of $\underline{N}$ currents of sinusoidal wave-form and the same frequency, having equal maximum values and displaced in time phase successively from one another by $1/\underline{N}$ of a period constitutes a symmetrical polyphase system of currents. Such a system of currents can be represented by a system of $\underline{N}$ harmonic vectors, having equal lengths and the same angular velocity, radiating from a point and equally spaced over an angle of 2π radians ~~//~~ with a displacement of $2\pi/\underline{N}$ radians between successive vectors.

A similar definition applies to any other quantities, such as voltages, which may be represented by harmonic vectors under similar conditions. The N currents flow in, or the N voltages are measured across, the N sections of which the polyphase circuit is composed. These sections, called for convenience the phases, are numbered from 1 to N to correspond to the symmetrical system of currents or voltages under consideration.

It follows from Equation 3 that the operator $1.\epsilon^{j\phi}$ will rotate a vector through an angle ϕ in the counterclockwise or positive direction without alteration of its magnitude. Thus take $\phi = k\frac{2\pi}{N}$, where $k = 1,2,3, \ldots N$; then $1.\epsilon^{jk\frac{2\pi}{N}}$ rotates a given vector through a positive angle of $k\frac{2\pi}{N}$. Writing the values of the N operators both in exponential and in component form, and taking for granted the unity tensor factor,

$$\epsilon^{j\frac{2\pi}{N}} = \cos\frac{2\pi}{N} + j.\sin\frac{2\pi}{N} = \lambda$$

$$\epsilon^{j\frac{4\pi}{N}} = \cos\frac{4\pi}{N} + j.\sin\frac{4\pi}{N} = \lambda^2$$

$$\epsilon^{j\frac{6\pi}{N}} = \cos\frac{6\pi}{N} + j.\sin\frac{6\pi}{N} = \lambda^3$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\epsilon^{j2\pi} = \cos 2\pi + j.\sin 2\pi = \lambda^N = 1,$$

are operators rotating a vector through counterclockwise angles of $2\pi/N$, $4\pi/N$, $6\pi/N$, ... 2π respectively, equivalent in effect to successive operation with λ, the operator producing rotation of $2\pi/N$.

The numerical identity of these operators must now be established. Remembering the numerical property of j, it is ~~is obvious that Remembering the numerical property of j, it~~ obvious that

$$1 = \epsilon^{jk2\pi} = \cos 2k\pi + j.\sin 2k\pi.$$

The Nth. root of unity is thus

$$1^{\frac{1}{N}} = \epsilon^{jk\frac{2\pi}{N}} = \cos 2k\frac{\pi}{N} + j.\sin 2k\frac{\pi}{N},$$

and there will clearly be N different Nth. roots. It follows, therefore, that by interpreting operation with j as numerically equivalent to multiplication with $\sqrt{-1}$, the N operators above defined are numerically equivalent to appropriate multiplications by the Nth. roots of unity. The values of the operators are given in Table I for values of N usual in polyphase theory.

Table I.

N	Operators, $1, \lambda, \lambda^2, \ldots \lambda^{N-1}$.
2	$1, -1$.
3	$1,\ -\frac{1}{2}+j\frac{\sqrt{3}}{2},\ -\frac{1}{2}-j\frac{\sqrt{3}}{2}$.
4	$1,\ j, -1,\ -j$.
6	$1,\ \frac{1}{2}+j\frac{\sqrt{3}}{2},\ -\frac{1}{2}+j\frac{\sqrt{3}}{2},\ -1,\ -\frac{1}{2}-j\frac{\sqrt{3}}{2},\ \frac{1}{2}-j\frac{\sqrt{3}}{2}$.

Now let i be an harmonic vector, representing the current in the first phase of a polyphase circuit; then the sequence

$$1i, \lambda i, \lambda^2 i, \lambda^3 i, \ldots \lambda^{N-1} i,$$

is a symmetrical polyphase system of N equal vectors numbered round the vector diagram in a counterclockwise direction; i.e. in negative phase sequence. The N currents attain their maxima in descending numerical order of the phases, as is shown by the typical instances given in Fig.4.

Reverting again to Equation 3 the operator $\epsilon^{-jk\frac{2\pi}{N}}$ will rotate a vector operand through an angle $k\frac{2\pi}{N}$ in a clockwise or negative direction without alteration of its magnitude. Now by ~~sim~~ simple trigonometry,

$$\epsilon^{-jk\frac{2\pi}{N}} = \cos k\frac{2\pi}{N} - j.\sin k\frac{2\pi}{N}$$

$$= \cos(N-k)\frac{2\pi}{N} + j.\sin(N-k)\frac{2\pi}{N}$$

$$= \epsilon^{j(N-k)\frac{2\pi}{N}}$$
$$= \left(\epsilon^{j\frac{2\pi}{N}}\right)^{(N-k)}$$
$$= \lambda^{N-k}.$$

Hence the sequence

$$1\underline{i},\ \lambda^{N-1}\underline{i},\ \lambda^{N-2}\underline{i},\ \lambda^{N-3}\underline{i},\ \ldots\ldots\lambda\underline{i},$$

is a symmetrical polyphase system of $\underline{N}$ equal vectors numbered round the vector diagram in a clockwise direction; i.e. in <u>positive sequence</u>. Cases of practical interest are shown in Fig.5; thses diagrams should be compared with the corresponding examples of negative sequense given in Fig.4. The positive sequence of vectors is the one usually employed in polyphase theory; successive ~~phase~~ currents attain their maxima in ascending numerical order of the phases.

The following property of a symmetrical polyphase system, whether of positive or negative sequence, is of very great importance. Remembering that $\lambda^N = 1$, the sum of the $\underline{N}$ vectors of the system is

$$(1+\lambda+\lambda^2+\ldots\ldots+\lambda^{N-1})\underline{i} = (\lambda+\lambda^2+\lambda^3\ldots\ldots\lambda^{N-1}+\lambda^N)\underline{i}$$
$$= \lambda(1+\lambda+\lambda^2+\ldots\ldots\lambda^{N-1})\underline{i}.$$

For this identity to be true either $\lambda = 1$, which is never the case, or else the bracketted term is zero; thus

$$(1+\lambda+\lambda^2+\ldots\ldots+\lambda^{N-1})\underline{i} = 0. \qquad (4.$$

The sum of the currents in a symmetrical polyphase system is therefore zero, the $\underline{N}$ vectors drawn end to end forming a closed polygon, as is obvious from the nature of the vector diagrams (see Figs. 4 and 5.)

The same property is also true of a symmetrical system of voltages.

It is because of the fact that the total voltage induced in the windings of an N phase alternator is zero that it is possible to interlink the phases in mesh connection without the appearance of any circulating current of fundamental frequency, as will be further pointed out in Section 6. Moreover, if the coils of a closed d.c. armature winding be treated as the phases of a polyphase system in which N is the number of coils it is on account of the property of a symmetrical system expressed by Equation 4 that it is possible to use such a closed winding in a machine with a sinusoidally distributed flux without an internal circulating current being produced. This theorem is the logical basis of the theory of armature windings set forth by Professor Heather in his paper.

5. THE VECTOR REPRESENTATION OF HARMONICS IN POLYPHASE CIRCUITS.

The results deduced in the last section apply primarily to sinusoidally varying quantities, but are immediately capable of extension to cover the case of complex wave-forms such as regularly occur in practice. Let all of the N currents (or voltages) have the same wave-form, amplitudes, and frequency, the waves being displaced in time-phase successively from one another by 1/N of a period. Then if the waves are analysed into their Fourier components, it is clear that the N fundamentals of the waves form a symmetrical N-phase system of sinusoidal quantities and may be at once represented by the preceding vector analysis. It remains now to show how the methods may be adapted to deal with the whole range of harmonics into which the waves have been assumed to have been decomposed.

If $2\pi/N$ be the phase displacement between the successive fundamentals, the displacement between successive nth. harmonics in the phases will be $n.2\pi/N$, n being the order of the harmonic. In vector notation any given harmonic can be represented by a harmonic vector $\underline{i}_n$ of length i_n rotating with angular velocity $n\omega$ as shown in Fig.6.

From the preceding section it is clear that the operator $\epsilon^{jkn\frac{2\pi}{N}}$ will rotate a vector $\underline{i}_n$ through a counterclockwise angle of $kn.2\pi/N$, so that the resulting vector can be taken to symbolise the nth. harmonic in the kth. phase. Now

$$\epsilon^{jkn\frac{2\pi}{N}} = (\epsilon^{jk\frac{2\pi}{N}})^n = (\lambda^k)^n = \lambda^{kn}.$$

Hence the sequence of vectors

$$1\underline{i}_n, \lambda^n\underline{i}_n, \lambda^{2n}\underline{i}_n, \lambda^{3n}\underline{i}_n, \ldots\ldots \lambda^{n(N-1)}\underline{i}_n,$$

represents the nth. harmonic in the N-phase system, the vectors being numbered in a counterclockwise direction round the vector diagram, i.e. in negative sequence. As it is usual in polyphase theory, as mentioned in Section 4, to treat the fundamental (n = 1) in positive sequence, it follows by a process similar to that adopted therein that the positive sequence for the nth. harmonic is

$$1\underline{i}_n, \lambda^{n(N-1)}\underline{i}_n, \lambda^{n(N-2)}\underline{i}_n, \lambda^{n(N-3)}\underline{i}_n, \ldots\ldots \lambda^n\underline{i}_n.$$

Adding together the vectors in the diagram for the nth. harmonic gives, since $\lambda^{nN} = 1$,

$$(1+\lambda^n+\lambda^{2n}+\lambda^{3n}+\ldots\lambda^{n(N-1)})\underline{i}_n = (\lambda^n+\lambda^{2n}+\lambda^{3n}+\ldots\lambda^{n(N-1)}+\lambda^{nN})\underline{i}_n$$
$$=\lambda^n(1+\lambda^n+\lambda^{2n}+\ldots\lambda^{n(N-1)})\underline{i}_n.$$

For this identity to be true, either

$$(1+\lambda^n+\lambda^{2n}+\ldots\lambda^{n(N-1)})\underline{i}_n = 0,$$

or

$$\lambda^n\,\underline{i}_n = 1\underline{i}_n.$$

Examining the ~~first~~ second possibility,

$$\lambda^n = \epsilon^{jn\frac{2\pi}{N}} = \cos \underline{n}\frac{2\pi}{N} + \underline{j}.\sin N\frac{2\pi}{N},$$

which to equal unity necessitates $\underline{n} = \underline{kN}$. Thus the harmonics fall into two groups, as follows;

a) If $\underline{n} = \underline{kN}$, *i.e.* if the order of the harmonic is a multiple of the number of phases, the $\underline{N}$ vectors are coincident and their sum is $\underline{N}\underline{i}_n$.

b) If $\underline{n} \neq \underline{kN}$, *i.e.* if the order of the harmonic is not a multiple of the number of phases, the $\underline{N}$ vectors when added together have zero resultant and form a closed polygon. Harmonics of these orders are represented by symmetrical $\underline{N}$-phase systems of vectors in positive or negative phase sequence, according to the value of $\underline{n}$, rotating with the appropriate angular velocity $\underline{n}\omega$.

These deductions apply equally to currents and to voltages.

<u>Harmonics in the Three-phase System</u>.

It is of particular practical interest to examine by this method the behaviour of the possible current and voltage harmonics in a three-phase circuit, assuming for the present that the phases of the circuit are independent and not interconnected in any way. It is very usual in practice for the harmonics, whether of current or voltage, to be of odd orders; cases are known, however, in which even terms exist so that the following deductions are made to include both odd and even harmonics.

Remembering that the operator λ rotates a vector through a positive angle of $2\pi/N$ radians, let $\lambda = \epsilon^{j\frac{2\pi}{3}}$ be now taken specially to denote the operator producing rotation of $2\pi/3$. Then for the nth. harmonic the positive sequence of the vectors is

$$1i_n,\ \lambda^{2n} i_n,\ \lambda^{n} i_n,$$

i_n being the vector of the nth. harmonic in the first or datum phase of the circuit, see Fig.7. All possible harmonics are comprised in the sequence of numbers derived from the expressions $3k+1$, $3k-1$, and $3k$, k being any integer from 0 upwards.

a) $n = 3k+1$. This gives $n = 1,4,7,10,13,16,\ldots$ and the vectors are $1i_n,\ \lambda^{6k+2} i_n,\ \lambda^{3k+1} i_n$. Now from the value of λ,

$$\lambda^{6k} = \epsilon^{jk4\pi} = \cos 4k\pi + j.\sin 4k\pi = 1$$

$$\lambda^{3k} = \epsilon^{jk2\pi} = \cos 2k\pi + j.\sin 2k\pi = 1.$$

Hence harmonics of the orders $n = 3k+1$ are represented by the vectors $1i_n,\ \lambda^{2} i_n,\ \lambda i_n$, and form symmetrical three-phase systems in positive sequence with pulsatance $(3k+1)\omega$

b) $n = 3k-1$. This gives $n = 2,5,8,11,14,\ldots$ and the vectors are $1i_n,\ \lambda^{6k-2} i_n,\ \lambda^{3k-1} i_n$. Inserting the values of λ^{6k} and λ^{3k}, and remembering that $\lambda^{-2} = \lambda$ and $\lambda^{-1} = \lambda^{2}$ in the case of the three-phase system, shows that harmonics of the orders $n = 3k-1$ are represented by the vectors $1i_n,\ \lambda i_n,\ \lambda^{2} i_n$, and form symmetrical three-phase systems in negative sequence with pulsatance $(3k-1)\omega$.

c) $n = 3k$. This gives $n = 3,6,9,12, \ldots$ and the vectors are $1i_n, \lambda^{6k} i_n, \lambda^{3k} i_n$. Hence harmonics of the orders $n = 3k$ are represented by the coincident vectors i_n, i_n, i_n, and form a zero sequence or single-phase system superposed in the circuit.

These well-known* facts are summarised in Fig.7, showing that harmonics of the orders $n = 3k+1$ behave in the circuit in a way precisely similar to the fundamental, but at $3k+1$ times fundamental pulsatance; harmonics of the orders $n = 3k-1$ behave in opposite sequence to the fundamental, but at $3k-1$ times fundamental pulsatance; and harmonics of the orders $n = 3k$ act like a single-phase system of $3k$ times fundamental pulsatance superposed on the polyphase circuit. These deductions are equally true of the currents and the voltages in the circuit.

6. THE VECTOR TREATMENT OF HARMONICS IN INTERLINKED POLYPHASE CIRCUITS.

In Section 5 it has been supposed that the several phases of which the polyphase circuit is composed are independent. It is well-known, however, that many important technical advantages are gained by interlinking the phases in star or in mesh; it is of particular interest, therefore, to examine the effect of such interlinkage on the voltages and currents in the polyphase circuit.

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* See for example, B. Hague, "Pressure harmonics in polyphase systems and windings", *Electrician*, vol.78, pp.710-712, 740-743, 765-769, 1917, where these and related matters, to be dealt with in the next section, are fully discussed by the usual analytical methods.

1. Star Connection. Voltages.

In the star connection of a polyphase circuit, *e.g.*, the windings of an alternator, the starting ends of the phases are joined together, forming a star point. Each finishing end is joined to a line, so that in an *N*-phase circuit there will be *N* lines conveying the energy in the circuit. An additional wire, called a star lead (sometimes referred to as a neutral lead) may be joined to the star point, bringing the total possible number of leads to *N* + 1.

Let the voltage harmonic of the order *n* in the successive phases of an *N*-phase circuit be represented by the sequence of *N* harmonic vectors

$$1\underline{e}_n, \lambda^{n(N-1)}\underline{e}_n, \lambda^{n(N-2)}\underline{e}_n, \ldots \lambda^{n(N-m-1)}\underline{e}_n, \ldots \lambda^{n}\underline{e}_n,$$

$\underline{e}_n$ being the vector representing this harmonic in the first or datum phase. These voltages are measured between the star point and the lines leading from the phases.

The voltage in the *m*th. phases is $\lambda^{n(N-m-1)}\underline{e}_n$, and that in the *m*+1 th. phase $\lambda^{n(N-m)}\underline{e}_n$. The vector difference between these two voltages will be the voltage across the lines joined to the free extremities of phases *m* and *m*+1, that is

$$\lambda^{n(N-m-1)}\underline{e}_n - \lambda^{n(N-m)}\underline{e}_n = \lambda^{n(N-m-1)}(1-\lambda^{-n})\underline{e}_n.$$

The first operator in this expression is the same as that occurring in the term in the previous sequence for the voltage in the *m*th. phase. Hence the *N* line voltages form an *N*-phase system of the same kind as the phase voltages. The factor in brackets, being independent of *m*, determines the magnitude and phase of the line voltages relative to the system of phase voltages. Now,

$$
\begin{aligned}
1-\lambda^{-n} = 1-\epsilon^{-jn\frac{2\pi}{N}} &= 1-(\cos \underline{n}\tfrac{2\pi}{N} - \underline{j}.\sin \underline{n}\tfrac{2\pi}{N}) \\
&= 1-\cos \underline{n}\tfrac{2\pi}{N} + \underline{j}.\sin \underline{n}\tfrac{2\pi}{N} \\
&= 2\sin \underline{n}\tfrac{\pi}{N}(\sin \underline{n}\tfrac{\pi}{N} + \underline{j}.\cos \underline{n}\tfrac{\pi}{N}) \\
&= 2\sin \underline{n}\tfrac{\pi}{N}\left[\cos(\tfrac{\pi}{2}-\underline{n}\tfrac{\pi}{N}) + \underline{j}.\sin(\tfrac{\pi}{2}-\underline{n}\tfrac{\pi}{N})\right] \\
&= 2\sin \underline{n}\tfrac{\pi}{N}.\epsilon^{j(\frac{\pi}{2}-n\frac{\pi}{N})} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (5
\end{aligned}
$$

The line voltages in an $\underline{N}$-phase circuit are therefore represented by the sequence of harmonic vectors

$$2\sin \underline{n}\tfrac{\pi}{N}.\epsilon^{j(\frac{\pi}{2}-n\frac{\pi}{N})}(1\underline{e_n}, \lambda^{n(N-1)}\underline{e_n}, \lambda^{n(N-2)}\underline{e_n}, \; . \; . \; . \; . \; \lambda^{n(N-m-1)}\underline{e_n}, \; . \; . \; . \; . \; \lambda^{n}\underline{e_n}).$$

Hence the line voltages are $2\sin \underline{n}\frac{\pi}{N}$ times as large as the phase voltages, have the same sequence, and are advanced relative to the phase voltages by a time displacement of $\frac{\pi}{2}-\underline{n}\frac{\pi}{N}$. Harmonics of the orders $\underline{n} \neq \underline{kN}$ form symmetrical systems in positive or negative sequence, according to the value of $\underline{n}$; while harmonics of the orders $\underline{n}=\underline{kN}$ vanish from the line voltages, since then $\sin \underline{n}\frac{\pi}{N} = \sin \underline{k}\pi = 0$.

__Voltages in the three-phase system.__ Equation 5 will now be applied to the three groups of harmonics distinguished in Section 5 in the three-phase system.

a) $\underline{n} = 3\underline{k}+1$. Phase voltages of these orders form symmetrical three-phase systems in positive sequence. Substituting $n = 3k+1$ makes

$$
\begin{aligned}
1-\lambda^{-n} &= 2\sin(3k+1)\tfrac{\pi}{3}.\epsilon^{j[\frac{\pi}{2}-(3k+1)\frac{\pi}{3}]} = 2\sin\tfrac{\pi}{3}.\cos \underline{k}\pi.\epsilon^{j\frac{\pi}{6}}.\epsilon^{-jk\pi} \\
&= 2\sin\tfrac{\pi}{3}.\cos^2 \underline{k}\pi.\epsilon^{j\frac{\pi}{6}} = 2\sin\tfrac{\pi}{3}.\epsilon^{j\frac{\pi}{6}} \\
&= \sqrt{3}.\epsilon^{j\frac{\pi}{6}},
\end{aligned}
$$

since $\epsilon^{-jk\pi} = \cos \underline{k}\pi$, and $\cos^2 \underline{k}\pi = 1$. Hence the line voltages of orders $\underline{n} = 3\underline{k}+1$ form symmetrical three-phase systems of vectors in positive sequence, $\sqrt{3}$ times the size of the phase voltage vectors and advanced ahead of the latter by an angle $\pi/6$, see Fig.8a.

b) $\underline{n} = 3\underline{k}-1$. Phase voltages of these orders form symmetrical three-phase systems in negative sequence. Substituting for $\underline{n} = 3\underline{k}-1$ makes

$$1-\lambda^{-n} = 2\sin(3k-1)\frac{\pi}{3}\,.\,\epsilon^{j[\frac{\pi}{2}-(3k-1)\frac{\pi}{3}]} = -2\sin\frac{\pi}{3}.\cos\underline{k}\pi.\epsilon^{j\frac{5\pi}{6}}.\epsilon^{-jk\pi}$$
$$= -2\sin\frac{\pi}{3}.\cos^2\underline{k}\pi.\epsilon^{j\pi}.\epsilon^{-j\frac{\pi}{6}} = 2\sin\frac{\pi}{3}.\epsilon^{-j\frac{\pi}{6}}$$
$$= \sqrt{3}.\epsilon^{-j\frac{\pi}{6}}.$$

Hence the line voltages of orders $\underline{n} = 3\underline{k}-1$ form symmetrical three-phase systems of vectors in negative sequence, $\sqrt{3}$ times the size of the phase voltage vectors and retarded behind the latter by an angle $\pi/6$, see Fig.8b.

c) $\underline{n} = 3\underline{k}$. Since in general $\sin\underline{n}\frac{\pi}{3} = 0$ when $\underline{n} = 3\underline{k}$ and N=3, these harmonics in the phases vanish from the line voltages when the phases are star connected, see Fig.8c.

2. <u>Star Connection</u>. <u>Currents</u>.

It is obvious from the circuit diagram of a star connected system that the current in a line is the same in magnitude and time-phase as the current in the phase to which the line is connected. In an $\underline{N}$ wire circuit the line and phase currents contain only those harmonics for which $n \neq \underline{kN}$. Since voltage harmonics of the orders $\underline{n} = \underline{kN}$ do not exist across the lines, currents of these orders cannot appear in the lines or the phases. In an $\underline{N}+1$ wire circuit, i.e., where the star lead is also present, since the $\underline{n} \neq \underline{kN}$ harmonics form symmetrical polyphase systems in which the instantaneous sum of the currents is zero, it follows that the N lines are, as before, necessary and sufficient to carry these harmonics. The $n \neq kN$ harmonics therefore floy entirely in the lines and do not appear in the star lead. Current harmonics of the orders $n = kN$ are of zero sequence, that is they are equal in magnitude and are in the same time-phase in all the phases of the circuit. They are therefore directed outwards at a given instant in all the phases and complete their circuit inwards through the star lead. Hence the lines in an N+1 wire circuit carry current harmonics of all orders, but the star lead carries only the harmonics of orders $n = kN$. The magnitude of the kNth. harmonic in the star lead is clearly N times its value in the lines.

<u>Currents in the three-phase system.</u> The preceding deductions will now be illustrated by means of the three-phase case. Referring to Fig.9a the phase and line currents in the three-wire system are shown; note the entire absence of all harmonics of orders $n = 3k$.

Since the harmonics of orders $n = 3k+1$ and $n = 3k-1$ form symmetrical three-phase systems, their instantaneous sum is zero; three leads therefore suffice to convey them from the phases, one lead acting momentarily as the return for the other two. The addition of a fourth wire, as in Fig.9b, does not affect these harmonics but merely provides a path by which currents of orders $n = 3k$ urged by the phase voltages of these orders into the lines, can return to the generator.

These results are conveniently summarised as follows:-

System.	Phase Current.	Line Current.	Star Lead.
Three wire.	$3k+1, 3k-1$.	$3k+1, 3k-1$.	----
Four wire.	$3k+1, 3k-1, 3k$.	$3k+1, 3k-1, 3k$.	$3k$ only.

3. Mesh Connection. Currents.

In the mesh connection of a polyphase circuit, such as the windings of a polyphase alternator, the finishing end of the first phase is joined to the starting end of the second, and so-on cyclically until the N phases form a closed mesh. A line is taken from each point of junction, making N lines in all.

Let the current harmonic of order n in the successive phases of an N-phase circuit be represented by the sequence of N harmonic vectors

$$1\underline{i}_n, \lambda^{n(N-1)}\underline{i}_n, \lambda^{n(N-2)}\underline{i}_n, \ldots \lambda^{n(N-m-1)}\underline{i}_n, \ldots \lambda^{n}\underline{i}_n,$$

$\underline{i}_n$ being the vector representing this harmonic in the first or datum phase. These currents would be measured by ammeters inserted in the phases.

The current in the mth. phase is $\lambda^{n(N-m-1)}\underline{i}_n$, and that in the $m+1$th. phase $\lambda^{n(N-m)}\underline{i}_n$. The vector difference between these two currents is the current flowing in the line joined to the junction of the phases m and $m+1$, that is,

$$\lambda^{n(N-m-1)}\underline{i}_n - \lambda^{n(N-m)}\underline{i}_n = \lambda^{n(N-m-1)}(1-\lambda^{-n})\underline{i}_n.$$

Making the transformation given by Equation 5 it is clear that the line currents in an N-phase circuit are represented by the sequence of harmonic vectors

$$2\sin\underline{n}\tfrac{\pi}{N}\cdot\epsilon^{j(\frac{\pi}{2}-n\frac{\pi}{N})}(1\underline{i}_n,\ \lambda^{n(N-1)}\underline{i}_n,\ \lambda^{n(N-2)}\underline{i}_n,\ \ldots\ \lambda^{n(N-m-1)}\underline{i}_n,\ \ldots\ \lambda^{n}\underline{i}_n).$$

Hence the line currents are $2\sin\underline{n}\frac{\pi}{N}$ times as large as the phase currents, have the same sequence, and are advanced relative to the phase currents by a time displacement $\frac{\pi}{2}-\underline{n}\frac{\pi}{N}$. Harmonics of the orders $n \neq k\underline{N}$ form symmetrical polyphase systems in positive or negative sequence, according to the value of $\underline{n}$; while harmonics of the orders $\underline{n} = \underline{kN}$ vanish entirely from the line currents, since $\sin\underline{n}\frac{\pi}{N} = \sin\underline{k}\pi = 0$. It follows, therefore that currents of the orders $\underline{n} = \underline{kN}$ circulate in the closed mesh.

Currents in the three-phase system. The behaviour of the three possible groups of current harmonics in a three-phase mesh connected circuit will now be examined.

a) $\underline{n} = 3\underline{k}+1$. Phase currents of these orders form symmetrical three-phase systems in positive sequence. From the general expression just deduced it follows that the line currents of orders $\underline{n} = 3\underline{k}+1$ also form symmetrical three-phase systems of vectors in positive sequence, $\sqrt{3}$ times the size of the phase current vectors and advanced ahead of the latter by an angle $\pi/6$, see Fig.10a.

b) $\underline{n} = 3\underline{k}-1$. Phase currents of these orders form symmetrical three-phase systems in negative sequence. From the general expression, therefore, the line currents of orders $\underline{n} = 3\underline{k}-1$ also form symmetrical three-phase systums of vectors in negative sequence, $\sqrt{3}$ times the size of the phase current vectors and retarded behind the latter by an angle $\pi/6$, see Fig.10b.

c) $n = 3k$. Since in general $\sin n\frac{\pi}{N} = 0$ when $n = 3k$ and $N=3$; these current harmonics do not appear in the lines, as shown in Fig.10c. As these harmonics are of zero sequence they are equal in magnitude and are in the same time-phase in each phase of the circuit; they are therefore added in simple series and circulate round the mesh, as in Fig.10.

4. Mesh Connection. Voltages.

Consider a mesh connected alternator. It has just been shown that the path of current harmonics of orders $n = kN$ is entirely within the closed mesh, round which these currents must be caused to circulate by the voltage harmonics of corresponding orders which are induced in the phases. Since voltages of orders $n = kN$ are of zero sequence they act in series round the mesh, and produce the circulating current. The magnitude of this current is such that the drop of voltage produced by it in flowing through the impedance of any phase is exactly equal to the electromotive force induced in the phase. Thus voltage harmonics of orders $n = kN$ induced in the phases are entirely short-circuited in the mesh and produce no potential difference across the lines. In this respect they act just like a number of equal cells connected to form a closed ring. A circulating current of such an amount will flow round the ring that the resistance drop through each cell exactly uses up the internal electromotive force of the cell. The total electromotive force round the ring is the sum of the individual e.m.f.s of the cells, but there will be no potential difference across the terminals of any cell.

Voltage harmonics of orders $n \neq kN$ form positive or negative sequence symmetrical N-phase systems. The voltages of these orders induced in any given phase are, therefore, at any instant equal and opposite to the sum of the voltages induced in the remaining phases, in consequence of the property of a symmetrical system given in Equation 4. Hence these harmonic voltages can produce no circulating current in the mesh, and therefore give rise to a potential difference across the lines.

Thus, the line voltages in an N-phase mesh connected circuit contain only harmonics for which $n \neq kN$, equal in magnitude, time-phase, and in the same sequence as the corresponding harmonics induced in the phases. Harmonics of orders $n = kN$ are absent from the line voltages, and are short-circuited in the mesh.

__Voltages in the three-phase system.__ These remarks will now be illustrated by application to the mesh connected, three-phase circuit shown in Fig. 11a.

Considering first the voltage harmonics of orders $n = 3k+1$ and $n = 3k-1$ induced in the phases, it is obvious that these appear at their full value across the lines. For instance, if at a given instant the voltage through phase I is directed from A to B, the sum of the the voltages in phases II and III will also be acting from A to B through these phases. This follows from the fact that these voltages form symmetrical three-phase systems, in which the sum of the three voltages is zero. Hence across the lines joined to A and B harmonics of the orders $n = 3k+1$ and $n = 3k-1$ appear as in phase I, and similarly for the other two pairs of lines.

Now it has been shown in the preseding section that currents of orders $n = 3k$ are necessarily confined to circulate in the mesh and cannot exist in the lines. The voltages of these orders are in zero sequence, equal in all the phases and directed at a given instant, for example, from A to B on phase I, B to C in phase II, and C to A in phase III. The current is of such strength-equal to the phase voltage divided by the phase impedance- that the induced voltage in each phase is used up in driving the current through the phase. Hence these harmonics produce no potential difference across the lines.

This property of the $n = 3k$ th. harmonics, often not properly understood by students, can be exactly imitated by the simple mesh connection of three cells, each of voltage E and resistance R, as shown in Fig. 11b. These cells conspire to send a current equal to E/R circulating round the mesh, the total electromotive force round which is $3E$. It is physically obvious, from a simple consideration of the diagram, that there can be no potential difference across lines joined to the points A, B, and C.

7. UNSYMMETRICAL POLYPHASE SYSTEMS AND THEIR SYMMETRICAL COORDINATES.

In Section 4 attention has been confined to symmetrical polyphase systems of currents and voltages of fundamental frequency and sinusoidal wave-form, represented by a number of equal radial vectors equally spaced round a circle, the vectors rotating with uniform angular velocity equal to the pulsatance. In many practical instances the currents - and sometimes also the voltages - in the phases of a polyphase circuit form an unbalanced or unsymmetrical system and are represented by a vector diagram in which the magnitudes and relative angular positions of the vectors have any desired values, determined by the manner in which the various phases are loaded. Since the commonly used term 'unbalanced' applied to such a system of currents is capable of a number of inconsistent meanings it is best to refer to the system as 'unsymmetrical', a term that renders apposite the method of treatment now to be described.

In Sections 5 and 6 the theorems of Section 4 have been extended to enable similar complex wave-forms of the current (or voltage) in the successive phases of an N-phase circuit to be dealt with by resolving the waves into their component Fourier harmonics. It was there shown that harmonics for which $n \neq kN$ are represented by symmetrical systems of harmonic vectors in positive or negative sequence, according to the value of n, rotating with n times the angular velocity of the vectors representing the fundamental. It follows, therefore, that any method now to be developed for unsymmetrical fundamentals applies a once to harmonics of these orders by mere substitution of the appropriate sequence and pulsatance. Again, it was shown that harmonics for which $\underline{n} = \underline{kN}$ form a zero sequence or single-phase system in the circuit, and are at once amenable to the usual single-phase theory, each harmonic being treated at its appropriate pulsatance. Attention will therefore be confined in this section to the discussion of the case of unsymmetrical fundamentals, the results of the preceding sections making the necessary extension to the treatment of harmonics sufficiently obvious.

The theory of unbalanced or unsymmetrical systems is, if approached in the usual way, of considerable analytical difficulty and the essential physical features of the solution are often obscured by the excess of mathematical detail. In any mathematical problem the labour can often be lightened and a more concise solution be obtained by the choice of suitable coordinates to which the problem can be referred. In a physical sense this means referring a given complex phenomenon to a series of simpler and similar phenomena which will represent it and are, as it were, its coordinates. For example, in the theory of the single-phase motor, the pulsating stator field is often expressed in terms of two rotating fields moving round the air-gap at uniform, equal, and opposite speeds. The uniform rotating fields are then the coordinates of the existing pulsating field. Or again, if energy is propagated along a transmission line by means of an alternating current a steady wave-motion is set up along the line. It is often simpler to refer the energy transmission to two travelling waves, one sent out from the alternator and the other, of different amplitude and phase, reflected back from the loaded end of the line. These two waves are then the coordinates of the original steady state on the line.

In a similar way it would appear that an unsymmetrical system of N coplanar vectors might be represented as the sum of a number of simple symmetrical systems. Fortescue* has

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*See Chas.L.Fortescue, loc.cit.ante. While the general treatment of the subject is due to this writer, particular instances of the principle have been dealt with by other workers. See, for example, L.V.Stokvis, "Sur la création des harmoniques 3 dans les alternateurs par suite des déséquilibrages des phases", Comptes Rendus, vol.159, pp.46-49, 1914. P.Müller, "Unsymmetrische Mehrphasensysteme", Elekt.Zeits., vol.39, pp.343-346, 353-356, 1918, in which the principle of symmetrical coordinates is applied to the graphical treatment of unbalanced two- and three-phase circuits, asynchronous motors, and phase converters. See also a paper by W.V.Lyon, El.World, vol.75, pp.1304-1308, 1920. The principle is clearly explained in its simple application to two and three-phase circuits in A.Hay, "Alternating Currents", 5th. edition, pp.375-380, 1923.

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shown that a system of N vectors of any length and relative phase displacement can be resolved into one system of N coincident vectors together with N-1 symmetrical N-phase systems, some of positive and some of negative sequence. These N systems are amenable to the theory just given in Section 4 and are called the Symmetrical Coordinates of the unsymmetrical system. If N is a prime number all the simple systems are different; if N is not prime, the coordinates are repeated in symmetrical groups. Further general development of the idea is possible and has been carried out by Fortescue in the paper cited. The particular utility of the method lies, however, in the simplicity that it introduces into the theory of the unsymmetrical three-phase system, which will be considered in the following section.

8. THE UNSYMMETRICAL THREE-PHASE SYSTEM.

Let $\underline{i}_{I}$, $\underline{i}_{II}$, $\underline{i}_{III}$, be any three current vectors in positive sequence. Then if λ be the operator rotating a vector through $2\pi/3$ it is easy to verify that, in virtue of the property of a symmetrical system proved in Equation 4 and since $\lambda^3 = 1$ and $\lambda^4 = \lambda$, the following identities are true:-

$$\left.\begin{aligned}
\underline{i}_{I} &= \tfrac{1}{3}(\underline{i}_{I}+\underline{i}_{II}+\underline{i}_{III}) + \tfrac{1}{3}(\underline{i}_{I}+\lambda\underline{i}_{II}+\lambda^2\underline{i}_{III}) + \tfrac{1}{3}(\underline{i}_{I}+\lambda^2\underline{i}_{II}+\lambda\underline{i}_{III}),\\
\underline{i}_{II} &= \tfrac{1}{3}(\underline{i}_{I}+\underline{i}_{II}+\underline{i}_{III}) + \tfrac{1}{3}\lambda^2(\underline{i}_{I}+\lambda\underline{i}_{II}+\lambda^2\underline{i}_{III}) + \tfrac{1}{3}\lambda(\underline{i}_{I}+\lambda^2\underline{i}_{II}+\lambda\underline{i}_{III}),\\
\underline{i}_{III} &= \tfrac{1}{3}(\underline{i}_{I}+\underline{i}_{II}+\underline{i}_{III}) + \tfrac{1}{3}\lambda(\underline{i}_{I}+\lambda\underline{i}_{II}+\lambda^2\underline{i}_{III}) + \tfrac{1}{3}\lambda^2(\underline{i}_{I}+\lambda^2\underline{i}_{II}+\lambda\underline{i}_{III}).
\end{aligned}\right\} \quad \ldots \; 6a$$

Now write

$$\left.\begin{aligned}
\underline{i}_{a} &= \tfrac{1}{3}(\underline{i}_{I}+\underline{i}_{II}+\underline{i}_{III}),\\
\underline{i}_{b} &= \tfrac{1}{3}(\underline{i}_{I}+\lambda\underline{i}_{II}+\lambda^2\underline{i}_{III}),\\
\underline{i}_{c} &= \tfrac{1}{3}(\underline{i}_{I}+\lambda^2\underline{i}_{II}+\lambda\underline{i}_{III});
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \; 6b$$

then

$$\left.\begin{aligned}
\underline{i}_{I} &= \underline{i}_{a}+\underline{i}_{b}+\underline{i}_{c},\\
\underline{i}_{II} &= \underline{i}_{a}+\lambda^2\underline{i}_{b}+\lambda\underline{i}_{c},\\
\underline{i}_{III} &= \underline{i}_{a}+\lambda\underline{i}_{b}+\lambda^2\underline{i}_{c}.
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \; 6c$$

Hence the three coplanar vectors $\underline{i}_{I}$, $\underline{i}_{II}$, and $\underline{i}_{III}$ can be represented by:-

(i). Three equal vectors $\underline{i}_{a}$, $\underline{i}_{a}$, $\underline{i}_{a}$, each equal to 1/3 of the sum of $\underline{i}_{I}$, $\underline{i}_{II}$, $\underline{i}_{III}$;

(ii). A symmetrical three-phase system $\underline{i}_{b}$, $\lambda^2\underline{i}_{b}$, $\lambda\underline{i}_{b}$, of positive phase sequence; and

(iii). A symmetrical three-phase system $\underline{i}_{c}$, $\lambda\underline{i}_{c}$, $\lambda^2\underline{i}_{c}$, of negati tive phase sequence.

It is obvious that precisely similar relations can be deduced in a corresponding way for unsymmetrical voltages.

In general, therefore, the properties of any unsymmetrical system of three-phase currents flowing in a circuit follow from superposition in the circuit of a single-phase system of three equal currents and two symmetrical three-phase systems of currents, one of positive and one of negative sequence. In a four-wire circuit the single-phase or zero sequence currents flow in the star connection, and divide at the star point into three equal parts, i_α, in the phases. In addition, both the positive and negative sequence currents are also present. In a three-wire circuit, whether star or mesh connected, $i_{I} + i_{II} + i_{III}$ is zero, by Kirchhoff's rule, and hence there is no zero sequence current. In this instance the unbalanced system is represented by a positive and a negative sequence current system superposed in the circuit. If the load becomes "balanced" or symmetrical and has positive sequence, then, $i_b = i_{I}$ and $i_c = 0$; i.e., the zero and negative sequence currents vanish, as would be expected.

The geometric significance of these equations can be most clearly appreciated by considering typical vector diagrams. Fig. 12 shows an unbalanced system of currents, such as would occur in a four-wire circuit in which $i_{I} + i_{II} + i_{III} \neq 0$, and its symmetrical coordinates. Fig. 13 is the corresponding diagram for a three-wire circuit where the three vectors necessarily have a zero sum.

The method of drawing these diagrams is very simple and follows at once from the geometric interpretation of the vector Equations 6a, b, and c. Briefly summarised the determination of the symmetrical coordinates of the vectors $\underline{i}_{I}, \underline{i}_{II}, \underline{i}_{III}$, is as follows:-

i). Draw the vectors $\underline{i}_{I}, \underline{i}_{II}, \underline{i}_{III}$ and trisect them.

ii). Draw $\frac{1}{3}\lambda\underline{i}_{II}, \frac{1}{3}\lambda^2\underline{i}_{II}, \frac{1}{3}\lambda\underline{i}_{III}, \frac{1}{3}\lambda^2\underline{i}_{III}$, shown in chain dotted lines; remembering that λ rotates a vector through a counterclockwise angle of $2\pi/3$.

iii). Find the vector sums corresponding to Equations 6b, thus determining $\underline{i}_a, \underline{i}_b, \underline{i}_c$; the construction is shown by the dotted lines.

iv). Finally draw the vectors $\lambda\underline{i}_b, \lambda^2\underline{i}_b, \lambda\underline{i}_c$ and $\lambda^2\underline{i}_c$, to complete the symmetrical systems.

Scalar relations. In the practical analysis of unsymmetrical three-phase systems into their symmetrical coordinates, the vector notation of Equations 6 suffices in theoretical work. For the purpose of numerical calculations in which the magnitudes and time-phase relations of the coordinates are desired the resolution may be made graphically as described, or alternatively by the use of scalar expressions.

Taking $\underline{i}_{I}$ as a datum, as in Fig. 14, let the vectors $\underline{i}_{II}$ and $\underline{i}_{III}$ be respectively $\underline{r}_2$ and $\underline{r}_3$ times as long as $\underline{i}_{I}$, and let them lag thereon in time-phase by angles α_1 and α_2. Let $\underline{i}_a, \underline{i}_b, \underline{i}_c$ lag relative to $\underline{i}_{I}$ by angles $\alpha_a, \alpha_b, \alpha_c$. Then if the amplitude of $\underline{i}_{I}$ be taken as unity and its time variation is assumed proportional to $\cos\omega t$, the three vector equations of 6b become in scalar form,

$$\underline{i}_a\cos(\omega t-\alpha_a) = \tfrac{1}{3}\left[\cos\omega t + \underline{r}_2\cos(\omega t-\alpha_2) + \underline{r}_3\cos(\omega t-\alpha_3)\right],$$

$$\underline{i}_b\cos(\omega t-\alpha_b) = \tfrac{1}{3}\left[\cos\omega t + \underline{r}_2\cos(\omega t-\alpha_2+\tfrac{2\pi}{3}) + \underline{r}_3\cos(\omega t-\alpha_3+\tfrac{4\pi}{3})\right],$$

$$\underline{i}_c\cos(\omega t-\alpha_c) = \tfrac{1}{3}\left[\cos\omega t + \underline{r}_2\cos(\omega t-\alpha_2+\tfrac{4\pi}{3}) + \underline{r}_3\cos(\omega t-\alpha_3+\tfrac{2\pi}{3})\right],$$

wherein $\underline{i}_a, \underline{i}_b, \underline{i}_c$, are the amplitudes of i_a, i_b, i_c, for unit amplitude of i_1. Expanding the trigonometrical terms and comparing coefficients gives for the components of i_a, i_b, i_c, along and perpendicular to i_1 respectively

$$\left.\begin{aligned}\underline{i}_a\cos\alpha_a &= \tfrac{1}{3}\left[\underline{r}_2\cos\alpha_2+\underline{r}_3\cos\alpha_3+1\right]\\ \underline{i}_a\sin\alpha_a &= \tfrac{1}{3}\left[\underline{r}_2\sin\alpha_2+\underline{r}_3\sin\alpha_3\right]\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ (7a$$

$$\left.\begin{aligned}\underline{i}_b\cos\alpha_b &= \tfrac{1}{6}\left[2-\underline{r}_2(\cos\alpha_2-\sqrt{3}\sin\alpha_2)-\underline{r}_3(\cos\alpha_3+\sqrt{3}\sin\alpha_3)\right]\\ \underline{i}_b\sin\alpha_b &= -\tfrac{1}{6}\left[\underline{r}_2(\sin\alpha_2+\sqrt{3}\cos\alpha_2)+\underline{r}_3(\sin\alpha_3-\sqrt{3}\cos\alpha_3)\right]\end{aligned}\right\}\quad\ldots\ (7b$$

$$\left.\begin{aligned}\underline{i}_c\cos\alpha_c &= \tfrac{1}{6}\left[2-\underline{r}_2(\cos\alpha_2+\sqrt{3}\sin\alpha_2)-\underline{r}_3(\cos\alpha_3-\sqrt{3}\sin\alpha_3)\right]\\ \underline{i}_c\sin\alpha_c &= -\tfrac{1}{6}\left[\underline{r}_2(\sin\alpha_2-\sqrt{3}\cos\alpha_2)+\underline{r}_3(\sin\alpha_3+\sqrt{3}\cos\alpha_3)\right]\end{aligned}\right\}\quad\ldots\ (7c$$

Solving these pairs of equations for the amplitudes and time-phases of the coordinates of the original unsymmetrical system gives

$$\left.\begin{aligned}\underline{i}_a &= \tfrac{1}{3}\sqrt{\left[1+\underline{r}_2^2+\underline{r}_3^2+2\underline{r}_2\cos\alpha_2+2\underline{r}_3\cos\alpha_3+2\underline{r}_2\underline{r}_3\cos(\alpha_3-\alpha_2)\right]}\\ \tan\alpha_a &= \left[\underline{r}_2\sin\alpha_2+\underline{r}_3\sin\alpha_3\right]/\left[\underline{r}_2\cos\alpha_2+\underline{r}_3\cos\alpha_3+1\right]\end{aligned}\right\}\ .\ 7d$$

$$\left.\begin{aligned}\underline{i}_b &= \tfrac{1}{3}\sqrt{1+\underline{r}_2^2+\underline{r}_3^2-\underline{r}_2(\cos\alpha_2-\sqrt{3}\sin\alpha_2)-\underline{r}_3(\cos\alpha_3+\sqrt{3}\sin\alpha_3)-\underline{r}_2\underline{r}_3\{\cos(\alpha_3-\alpha_2)-\sqrt{3}\sin(\alpha_3-\alpha_2)\}}\\ \tan\alpha_b &= -\left[\underline{r}_2(\sin\alpha_2+\sqrt{3}\cos\alpha_2)+\underline{r}_3(\sin\alpha_3-\sqrt{3}\cos\alpha_3)\right]/\left[2-\underline{r}_2(\cos\alpha_2-\sqrt{3}\sin\alpha_2)-\underline{r}_3(\cos\alpha_3+\sqrt{3}\sin\alpha_3)\right]\end{aligned}\right\}\ .\ 7e$$

$$\left.\begin{aligned}\underline{i}_c &= \tfrac{1}{3}\sqrt{1+\underline{r}_2^2+\underline{r}_3^2-\underline{r}_2(\cos\alpha_2+\sqrt{3}\sin\alpha_2)-\underline{r}_3(\cos\alpha_3-\sqrt{3}\sin\alpha_3)-\underline{r}_2\underline{r}_3\{\cos(\alpha_3-\alpha_2)+\sqrt{3}\sin(\alpha_3-\alpha_2)\}}\\ \tan\alpha_c &= -\left[\underline{r}_2(\sin\alpha_2-\sqrt{3}\cos\alpha_2)+\underline{r}_3(\sin\alpha_3+\sqrt{3}\cos\alpha_3)\right]/\left[2-\underline{r}_2(\cos\alpha_2+\sqrt{3}\sin\alpha_2)-\underline{r}_3(\cos\alpha_3-\sqrt{3}\sin\alpha_3)\right]\end{aligned}\right\}\ .\ .7f$$

Vanishing of coordinates. The unbalance of a three-phase system is characterised by the appearance, in general, of a zero sequence and a negative sequence system of current in the circuit together with a positive sequence system. It will later be shown that the latter alone is responsible for the transmission of the electrical energy in the circuit; the zero and negative sequence components of the unbalance are wattless and have important effects in the alternator supplying the unbalanced load. It is of interest to examine the conditions under which the zero and negative sequence characteristics of the unbalance will be absent.

i). Zero sequence currents. The zero sequence or single-phase component of the unbalance will vanish if $\underline{i}_a = 0$, that is if $\underline{i}_I + \underline{i}_{II} + \underline{i}_{III} = 0$, from Equation 6b. Thus there will be no zero component in all cases in which the three current~~s~~ vectors form a closed triangle. This condition occurs automatically in a three-wire system and may be satisfied in a four-wire circuit by the proper choice of the currents. Expressed in scalar form, $\underline{i}_a$ will vanish if its two components are simultaneously zero; putting zero in both elements of Equations 7a the geometric condition for no single-phase current is

$$\left.\begin{aligned} \underline{r}_2 \cos\alpha_2 + \underline{r}_3 \cos\alpha_3 &= -1 \\ \underline{r}_2 \sin\alpha_2 + \underline{r}_3 \sin\alpha_3 &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (8a$$

ii). Negative sequence currents. The negative sequence component of unbalance will be non-existent if $\underline{i}_c = 0$, that is if $\underline{i}_I + \lambda^2 \underline{i}_{II} + \lambda \underline{i}_{III} = 0$, from Equation 6b. There will be no negative sequence component, therefore, in all cases where the three vectors $\underline{i}_I$, $\lambda^2 \underline{i}_{II}$, $\lambda \underline{i}_{III}$ form a closed triangle. In a three-wire circuit, since $\underline{i}_I + \underline{i}_{II} + \underline{i}_{III}$ is necessarily zero, this condition can only be satisfied when the three original currents form a symmetrical three-phase system. In a four-wire circuit, however, the condition can be fulfilled by an infinite number of possible arrangements of the original vectors. In a scalar form, putting both components of $\underline{i}_c$ equal to zero in Equation 7 gives

$$\left.\begin{aligned} \underline{r}_2(\cos\alpha_2+\sqrt{3}\sin\alpha_2) + \underline{r}_3(\cos\alpha_3 \mp \sqrt{3}\sin\alpha_3) &= 2 \\ \underline{r}_2(\sin\alpha_2-\sqrt{3}\cos\alpha_2) + \underline{r}_3(\sin\alpha_3+\sqrt{3}\cos\alpha_3) &= 0 \end{aligned}\right\} \quad . \; . \; (8b$$

to be simultaneously satisfied.

iii). Zero and negative sequence currents. The only condition that will simultaneously satisfy Equations 8a and b is that of symmetry among the original current vectors, that is $\underline{i}_{II} = \lambda^2 \underline{i}_I$ and $\underline{i}_{III} = \lambda \underline{i}_I$, or in scalar form $\underline{r}_2 = \underline{r}_3 = 1$, $\alpha_2 = 2\pi/3$, and $\alpha_3 = 4\pi/3$. Hence dissymmetry is always characterised by the presence of zero and negative sequence currents in a four-wire system and by the latter alone in a three-wire circuit.

9. HEMISYMMETRICAL SYSTEMS IN VECTOR NOTATION. THE QUARTER-PHASE SYSTEM. UNSYMMETRICAL TWO-PHASE SYSTEMS.

It remains to notice another type of polyphase system, originally treated by Steinmetz, which is defined in the following way. A system of N currents of sinusoidal wave-form and the same frequency, having equal maximum values and displaced in time-phase successively from one another by $1/N$ of a half-period constitutes a hemisymmetrical polyphase system of currents.

Such a system of currents can be represented by a system of $\underline{N}$ harmonic vectors, having equal lengths and the same angular velocity, radiating from a point and equally spaced over an angle of π radians with a phase displacement of $\pi/\underline{N}$ radians between successive vectors. It is obvious that the vector diagram of such a system is exactly half that of a symmetrical system of $2\underline{N}$ vectors, thus justifying the name of hemisymmetrical. Similar definitions clearly apply to voltages.

~~It fo~~ It follows from this definition that all hemisymmetrical $\underline{N}$-phase systems are necessarily unbalanced or unsymmetrical $\underline{N}$-phase systems and are immediateky capable of treatment by resolution into symmetrical coordinates, *i.e.*, a zero sequence system plus $\underline{N}$-1 symmetrical $\underline{N}$-phase systems.

Quarter-phase system. —— The only hemisymmetrical system of technical importance is the hemisymmetrical two-phase system represented by two equal vector separated by an angle of $\pi/2$. This is more shortly termed the "quarter-phase" system.

If $\underline{i}$ be the vector representing the current in the first or datum phase, a positive sequence quarter-phase system is given by the vectors

$$\underline{i}, -j\underline{i};$$

and a negative sequence system by

$$\underline{i}, j\underline{i},$$

both of which are shown in Fig, 15.

Unsymmetrical two-phase systems in symmetrical coordinates. —— It follows from the definition of a symmetrical system given in Section 4 that a symmetrical two-phase system of currents is represented by two equal vectors separated by an angle of π radians, *i.e.*, in opposition, see Figs. 4 and 5. An unsymmetrical two-phase system consists of any two vectors the lengths of which are, in general, unequal and the phase displacement between them is not necessarily π radians. It is clear that the quarter-phase system is a particular case of the unsymmetrical two-phase system, the vectors being equal in length but separated by $\pi/2$ radians.

It will now be shown that a pair of vectors $\underline{i}_I, \underline{i}_{II}$, in positive sequence may be expressed as the sum of a zero sequence of two vectors and a symmetrical two-phase system, the currents represented by them being superposed in the circuit. It is obvious that

$$\left.\begin{aligned} \underline{i}_I &= \tfrac{1}{2}(\underline{i}_I + \underline{i}_{II}) + \tfrac{1}{2}(\underline{i}_I - \underline{i}_{II}) = \underline{i}_a + \underline{i}_b \\ \underline{i}_{II} &= \tfrac{1}{2}(\underline{i}_I + \underline{i}_{II}) - \tfrac{1}{2}(\underline{i}_I - \underline{i}_{II}) = \underline{i}_a - \underline{i}_b \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (9a$$

where

$$\left.\begin{aligned} \underline{i}_a &= \tfrac{1}{2}(\underline{i}_I + \underline{i}_{II}) \\ \underline{i}_b &= \tfrac{1}{2}(\underline{i}_I - \underline{i}_{II}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9b$$

Hence two coplanar vectors $\underline{i}_I, \underline{i}_{II}$ can be represented by:-

(i). Two equal vectors $\underline{i}_a$ each equal to half the sum of $\underline{i}_I$ and $\underline{i}_{II}$; together with

(ii). A symmetrical two-phase system $\underline{i}_b, -\underline{i}_b$ where $\underline{i}_b$ is half the difference between $\underline{i}_I$ and $\underline{i}_{II}$.

An unsymmetrical system of currents represented by $\underline{i}_I$ and $\underline{i}_{II}$ is therefore equivalent to a single-phase or zero sequence system of two equal currents $(\underline{i}_a, \underline{i}_a)$ and a symmetrical two-phase system of currents superposed in the circuit $(\underline{i}_b, -\underline{i}_b)$. Fig. 16

16a has been drawn to show the resolution of a general two-phase system into its symmetrical coordinates. In a like manner Fig.17 illustrates the symmetrical coordinates of a quarter phase system. In this instance, since $\underline{i}_I = \underline{j}\underline{i}_{II}$, the values of $\underline{i}_a$ and $\underline{i}_b$ become $\underline{i}_a = \frac{1}{2}(1+\underline{j})\underline{i}_{II}$ and $\underline{i}_b = -\frac{1}{2}(1-\underline{j})\underline{i}_{II}$. Now

$$1 \pm \underline{j} = 1 + \cos\tfrac{\pi}{2} \pm \underline{j}.\sin\tfrac{\pi}{2} = 2.\cos^2\tfrac{\pi}{4} \pm \underline{j}.2.\sin\tfrac{\pi}{4}.\cos\tfrac{\pi}{4}$$
$$= 2.\cos\tfrac{\pi}{4}\left(\cos\tfrac{\pi}{4} \pm \underline{j}.\sin\tfrac{\pi}{4}\right)$$
$$= \sqrt{2}.\epsilon^{\pm j\frac{\pi}{4}}.$$

Hence for the quarter-phase system the coordinates are

$$\underline{i}_a = \frac{1}{\sqrt{2}}\epsilon^{j\frac{\pi}{4}}\underline{i}_{II},$$
$$\underline{i}_b = \frac{1}{\sqrt{2}}\epsilon^{-j\frac{\pi}{4}}\underline{i}_{II}.$$

The process of drawing Figs.16a and 17 is briefly as follows:-

i). Draw the vectors $\underline{i}_I, \underline{i}_{II}$, and bisect them.

ii). Draw $-\frac{1}{2}\underline{i}_{II}$, shown in chain-dotted line.

iii). Find the vector sums corresponding to Equations 9b, thus determining $\underline{i}_a$ and $\underline{i}_b$; the construction is shown by the dotted lines.

iv). Draw the vector $-\underline{i}_b$ to complete the symmetrical two-phase system.

<u>Scalar relations</u>.____ The scalar magnitudes and time-phases of the symmetrical coordinates of a given unsymmetrical two-phase system are easily found. Taking $\underline{i}_I$ as a datum, as in Fig. 16a, let the vector $\underline{i}_{II}$ be r_2 times as long as $\underline{i}_I$ and let its lag thereon be α_2. Let $\underline{i}_a, \underline{i}_b$ lag relative to $\underline{i}_I$ by angles α_a, α_b, their magnitudes being i_a and i_b. Assuming $\underline{i}_I$ to be of unit

amplitude and that its time variation is proportional to $\cos \omega \underline{t}$. Equations 9b become in scalar form,

$$\underline{i}_a \cos(\omega \underline{t} - \alpha_a) = \tfrac{1}{2}\left[\cos \omega \underline{t} + \underline{r}_2 \cos(\omega \underline{t} - \alpha_2)\right],$$

$$\underline{i}_b \cos(\omega \underline{t} - \alpha_b) = \tfrac{1}{2}\left[\cos \omega \underline{t} - \underline{r}_2 \cos(\omega \underline{t} - \alpha_2)\right].$$

Expanding and comparing the coefficients gives for the components of $\underline{i}_a, \underline{i}_b$ along and perpendicular to i_I respectively,

$$\left.\begin{aligned} \underline{i}_a \cos\alpha_a &= \tfrac{1}{2}(1 + \underline{r}_2 \cos\alpha_2) \\ \underline{i}_a \sin\alpha_a &= \tfrac{1}{2}\underline{r}_2 \sin\alpha_2 \end{aligned}\right\} \quad \text{(10a)}$$

$$\left.\begin{aligned} \underline{i}_b \cos\alpha_b &= \tfrac{1}{2}(1 - \underline{r}_2 \cos\alpha_2) \\ \underline{i}_b \sin\alpha_b &= -\tfrac{1}{2}\underline{r}_2 \sin\alpha_2 \end{aligned}\right\} \quad \text{(10b)}$$

Solving for the amplitudes and time-phases of the coordinates of the original system gives

$$\left.\begin{aligned} \underline{i}_a &= \tfrac{1}{2}\sqrt{1 + \underline{r}_2^2 + 2\underline{r}_2 \cos\alpha_2} \\ \tan\alpha_a &= \underline{r}_2 \sin\alpha_2/(1+\underline{r}_2 \cos\alpha_2) \end{aligned}\right\} \quad \text{(10c)}$$

$$\left.\begin{aligned} \underline{i}_b &= \tfrac{1}{2}\sqrt{1 + \underline{r}_2^2 - 2\underline{r}_2 \cos\alpha_2} \\ \tan\alpha_b &= -\underline{r}_2 \sin\alpha_2/(1-\underline{r}_2 \cos\alpha_2) \end{aligned}\right\} \quad \text{(10d)}$$

It is physically obvious, or readily deduced from Equations 10a, that the single-phase component will vanish only if $\underline{r}_2 = 1$ and $\alpha_2 = 180$ degrees, *i.e.*, when the original currents form a symmetrical two-phase system.

Unsymmetrical two-phase systems in quarter-phase coordinates. Unsymmetrical two-phase systems of currents usually occur in a circuit in which there is a quarter-phase system of voltages, *i.e.*, in an unbalanced quarter-phase circuit. It is of practical interest to determine if it is possible to express such an unsymmetrical system in terms of two quarter-phase systems superposed in the circuit.

As before, let i_{I}, i_{II} be any pair of vectors in positive sequence; then it is obvious that

$$\left.\begin{aligned} i_{I} &= \tfrac{1}{2}(i_{I} + j\cdot i_{II}) + \tfrac{1}{2}(i_{I} - j\cdot i_{II}) = i_{c} + i_{d} \\ i_{II} &= -\tfrac{1}{2}j(i_{I} + j\cdot i_{II}) + \tfrac{1}{2}j(i_{I} - j\cdot i_{II}) = -ji_{c} + ji_{d} \end{aligned}\right\} \quad \ldots\ldots (11a)$$

where

$$\left.\begin{aligned} i_{c} &= \tfrac{1}{2}(i_{I} + ji_{II}) \\ i_{d} &= \tfrac{1}{2}(i_{I} - ji_{II}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

Hence two coplanar vectors i_{I}, i_{II} can be represented by:-

i). A hemisymmetrical two-phase (quarter-phase) system i_{c}, $-ji_{c}$ of positive sequence; together with

ii). A hemisymmetrical two-phase (quarter-phase) system i_{d}, ji_{d} of negative sequence.

An unsymmetrical system of currents represented by i_{I} and i_{II} is therefore equivalent to two quarter-phase systems of currents superposed in the circuit, one of positive and the other of negative sequence. In Fig. 16b the vectors i_{I} and i_{II} of Fig 16a are resolved into quarter-phase coordinates in the manner represented by Equations 11a and b. The process of drawing this diagram is as follows:-

i). Draw the vectors i_{I} and i_{II} and bisect them.

ii). Draw $\tfrac{1}{2}ji_{II}$ and $-\tfrac{1}{2}ji_{II}$, shown by the chain-dotted lines.

iii). Find the vector sums corresponding to Equations 11b, thus determining i_{c} and i_{d}; the construction is shown in dotted lines.

v). Draw the vectors $-ji_{c}$ and ji_{d} to complete the quarter-phase systems.

Scalar relations. Let the amplitudes of $\underline{i}_c$ and $\underline{i}_d$ be $\underline{i}_c$ and $\underline{i}_d$ for a unit amplitude of $\underline{i}_r$; let them lag on $\underline{i}_r$ by angles α_c and α_d respectively. Then using the notation of the previous scalar equations

$$\underline{i}_c\cos(\omega\underline{t}-\alpha_c) = \tfrac{1}{2}\left[\cos\omega\underline{t} + \underline{r}_2\cos(\omega\underline{t}-\alpha_2+\tfrac{\pi}{2})\right]$$

$$\underline{i}_d\cos(\omega\underline{t}-\alpha_d) = \tfrac{1}{2}\left[\cos\omega\underline{t} + \underline{r}_2\cos(\omega\underline{t}-\alpha_2-\tfrac{\pi}{2})\right]$$

Expanding and comparing coefficients gives for the components of $\underline{i}_c, \underline{i}_d$ along and perpendicular to $\underline{i}_r$ respectively

$$\left.\begin{aligned}\underline{i}_c\cos\alpha_c &= \tfrac{1}{2}(1+\underline{r}_2\sin\alpha_2)\\ \underline{i}_c\sin\alpha_c &= -\tfrac{1}{2}\underline{r}_2\cos\alpha_2\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12a$$

$$\left.\begin{aligned}\underline{i}_d\cos\alpha_d &= \tfrac{1}{2}(1-\underline{r}_2\sin\alpha_2)\\ \underline{i}_d\sin\alpha_d &= \tfrac{1}{2}\underline{r}_2\cos\alpha_2\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12b$$

Solving for the magnitudes and phase-angles,

$$\left.\begin{aligned}\underline{i}_c &= \tfrac{1}{2}\sqrt{1 + \underline{r}_2^2 + 2\underline{r}_2\sin\alpha_2}\\ \tan\alpha_c &= -\underline{r}_2\cos\alpha_2/(1 + \underline{r}_2\sin\alpha_2)\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12c$$

$$\left.\begin{aligned}\underline{i}_d &= \tfrac{1}{2}\sqrt{1 + \underline{r}_2^2 - 2\underline{r}_2\sin\alpha_2}\\ \tan\alpha_d &= \underline{r}_2\cos\alpha_2/(1 - \underline{r}_2\sin\alpha_2)\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12d$$

It is physically obvious, or easily deduced from Equation 12b, that the negative sequence currents vanish when $\underline{r}_2 = 1$ and $\alpha_2 = \pi/2$, *i.e.*, when the original currents form a quarter-phase system.

10. POWER IN UNSYMMETRICAL SYSTEMS.

It is of the greatest practical importance to examine the way in which power is transmitted in an unsymmetrical polyphase circuit. Let it be supposed that the voltages in the phases form a symmetrical $\underline{N}$-phase system of vectors, but that the currents form an unsymmetrical system. Moreover, unless the internal characteristics of the supply alternator are very poor, the voltages will remain very closely symmetrical even though the phases are loaded unsymmetrically. To a high degree of approximation, therefore, the voltage dissymmetry can be taken as negligible in most pmactical instances. If voltage dissymmetry is present it can easily be treated by resolving the unsymmetrical voltage diagram into its symmetrical coordinates and dealing with the behaviour of each coordinate in turn with respect to the coordinates of the current diagram.

It will be assumed that fundamental time-variation only is involved. No lack of generality can ensue from this assumption, since harmonics can be treated at once by the substitution of the appropriate pulsatance and a knowledge of the proper phase sequence deduced in Sections 5 and 6 of this paper.

Now it has just been shown that an unsymmetrical current diagram can be resolved into the sum of $\underline{N}$ diagrams, viz., one representing a single-phase current flowing in each phase, and $\underline{N}$-1 symmetrical $\underline{N}$-phase systems some of positive and some of negative sequence. If the voltages are in positive sequence it is easy to show how the power in the circuit is allocated among the various symmetrical coordinates of the current system.

It should first be observed by comparison of Figs. 4 and 5 that the properties of a negative phase-sequence of vectors numbered counterclockwise round the counterclockwise rotating vector diagram are exactly represented by a series of vectors numbered clockwise round a clockwise rotating diagram, *i.e.*, by a positive sequence of vectors rotating in the opposite of the standard direction. Hence the reversal of phase sequence can be analytically treated as a reversal of the sign of ω.

In the *k*th. phase of a system in which the angular separation of the vectors is $\beta = 2\pi/N$, the voltage is

$$e_k = e_1 \cos\left[\omega t - (k-1)\beta\right] .$$

If there is a symmetrical system of current in the system, *e.g.*, one of the symmetrical coordinates of an unbalanced load, displaced by an angle ϕ from the voltage system as in Fig.18a, at the instant $t = 0$, the current in the *k*th. phase is

$$i_k = i_1\left\{\cos\left[\omega t - (k-1)\beta - \phi\right]\right\},$$

in which ω_1 has the same numerical value as ω but may be of positive or negative sign according to the sequence of the current system. If the current is of positive sequence the current and voltage diagrams rotate together in the counterclockwise direction but do not change their positions relative to one another. If the current is of negative sequence the current diagram rotates in the clockwise direction, and therefore moves its position relative to the counterclockwise rotating voltage diagram with an angular velocity 2ω.

The instantaneous power in the phase is

$$\begin{aligned} p_k = e_k i_k &= e_1 i_1 \cos\left[\omega t - (k-1)\beta\right].\cos\left[\omega t - (k-1)\beta - \phi\right] \\ &= EI\left\{\cos\left[(\omega+\omega_1)t - 2(k-1)\beta - \phi\right] + \cos\left[(\omega-\omega_1)t + \phi\right]\right., \end{aligned}$$

by the usual transformations, E and I being the virtual values of the voltage and current.

Positive sequence. If the currents form a system in positive sequence $\omega_1 = +\omega$ and the instantaneous power in the kth. phase is

$$p_k = EI\left\{\cos\phi + \cos\left[2\omega t - 2(k-1)\beta - \phi\right]\right\};$$

i.e., pulsates at double frequency about the average value $EI\cos\phi$.

The power in the entire N-phase system will be the sum of this expression taken over all values of k from 1 to N. Now

$$\sum_1^N \cos\left[2\omega t - 2(k-1)\beta - \phi\right] = \cos(2\omega t - \phi + \tfrac{2\pi}{N})\frac{\sin 2\pi}{\sin\frac{2\pi}{N}} = 0$$

if N is greater than 2. Hence the total instantaneous power due to a positive sequence system of voltages and a similar system of currents is steady and equal to

$$P = NEI\cos\phi \, .$$

The time-average of the total power over a period is thus also steady and equal to P, the instantaneous power in the system.

Negative sequence. If now the currents form a system in negative sequence $\omega_1 = -\omega$ and the instantaneous power in the kth. phase will be

$$p_k = EI\left\{\cos\left[\phi + 2(k-1)\beta\right] + \cos\left[2\omega t + \phi\right]\right\}$$

which again pulsates at double frequency in each phase of the system.

Summing, the total instantaneous power in the N-phase system is

$$P = NEI\cos\left[2\omega t + \phi\right],$$

which pulsates at double frequency.

It follows, therefore, that the time-average of the total power is zero;the negative sequence currents are therefore wattless with respect to the positive sequence system of voltages.

Zero sequence. If the currents are in zero sequence,as shown in Fig.18b,having equal values in all the phases given by

$$i_k = i_1 \cos(\omega t - \phi),$$

the instantaneous power in the kth, phase will be

$$p_k = e_1 i_1 \cos(\omega t - \phi).\cos[\omega t - (k-1)\beta]$$
$$= EI\{\cos[2\omega t - (k-1)\beta - \phi] + \cos[(k-1)\beta - \phi]\},$$

which pulsates at double frequency.

Summing for all the phases in the previous way makes the total instantaneous power in the system equal to zero. Hence also the time-average of the total power due to a positive sequence system of voltages and a zero sequence system of currents is zero. Such currents are,therefore,wattless.

If the positive,negative,and zero sequences of current just considered were the symmetrical coordinates of an unsymmetrical N-phase system of currents,the following important conclusion emerges. The entire energy in the circuit is transmitted by the positive sequence currents,the negative and zero sequence currents being wattless. Now it has been shown in the preceding sections that the negative sequence currents (and in the case of certain kinds of 'unbalance' the zero sequence currents also) are the characteristic feature of dissymmetry in a polyphase circuit. In a single-phase circuit,or in a completely symmetrical polyphase circuit the nature of the load is specified by the power-factor. In an unbalanced polyphase circuit the nature of the load requires for its complete statement the knowledge of the power-factor of the positive sequence currents and in addition the amount of the wattless negative and zero sequence currents.

The transmission of power by an unsymmetrical system of currents can be ckearly pictured in the following way. True power of amount NEIcosϕ is transmitted only by the positive sequence currents,I being the virtual value of these currents and ϕ their phase displacement relative to the positive sequence of voltages. The positive sequence currents are responsible for reactive power NEIsinϕ ,and so far the transmission is exactly like that in a perfectly balanced system carrying the positive sequence currents only. The negative sequence and zero sequence currents are,as demonstrated above,wattless and therefore apparently contribute a further amount to the reactive power in the circuit. The effect of unbalance is therefore to introduce into a circuit transmitting a given amount of power an increased amount of reactive power pulsating at double frequency. Hence so far as the station alternator is concerned,the effects of unbalance are precisely similar to that of an increased flow of reactive energy through its windings,with the customary bad effects on the regulation of the machine.

To put the matter in another way,suppose an alternator to supplying a certain amount of power to a circuit; the currents and voltages are supposed to be symmetrical systems with a certain phase-displacement, characterising the load, between them. Due to the reactive energy entering the machine a certain field excitation will be required to enable the alternator to maintain its volts, the excitation being fixed by the lagging or leading power-factor of the load. Now, keeping the power constant let the loads be unbalanced. Then the above theory shows that the positive sequence currents will supply all the power to the external load and its reactive component will call for the same field excitation as before. But, in addition, the unbalance, as distinct from the true reactance of the load, calls into being wattless negative and zero sequence currents which, in a broad view, act as an apparent increase in the load reactance since they are wattless. Inside the machine the general effect will be to influence still more the regulation and the required excitation as compared with what is found necessary for the same supply of power by a balanced load. Of the precise nature of the internal reactions in the machine consequent upon these negative and zero sequence currents a few words will be said in the following section.

11. TECHNICAL APPLICATIONS OF SYMMETRICAL COORDINATES.

Having described the properties of unsymmetrical systems of currents, particularly in the important practical case of the three-phase circuit, in terms of the theory of symmetrical coordinates it is now necessary briefly to state one or two instances in which this method of treatment leads to analytical simplicity, to clearer ideas, and often to new physical conceptions.

The practical designation of unbalance.___ Unsymmetrical loading of the phases of a supply system is as important to the station engineer as the influence of bad power-factor. For,as has been shown,unbalance causes an apparent increase in the reactive energy circulating through the alternator,with consequent effects upon the voltage regulation. To specify the magnitude of the dissymmetry in a practical way various suggesstions have been made,chiefly in the direction of stating an unbalance factor for the system. The theory of symmetrical coordinates has been of great assistance in this matter,and has enabled a Committee of the Americal Institute of Electrical Engineers to formulate certain definitions of considerable practical value.

The measurement of negative sequence current.____ In supplying three-phased energy to a consumer it is necessary to be able to measure,and if necessary to limit,the amount of the unbalance characteristic. Since most three-phase apparatus is supplied on the three-wire system,negative sequence currents characterise the unbalance and must be measured; moreover,if they become excessive the unbalanced circuit must be automatically disconnected by a relay trip worked by the negative sequence currents only. Referring to Equations 6b,in a three-wire system $\underline{i}_a = 0$,so that

$$\underline{i}_c = \frac{1}{3}\left[(\lambda^2-1)\underline{i}_{II} + (\lambda-1)\underline{i}_{III}\right].$$

It is not a difficult matter now to devise a network of impedances supplied from current transformers in lines II and

III such that one branch of the network carries the current $\underline{i}_c$; the impedances of the network, its mode of connection, and the arrangement of the transformers are such that the currents $(\lambda^2-1)\underline{i}_{II}$ and $(\lambda-1)\underline{i}_{III}$ are superposed in the chosen branch.* An

* See J.V.Breisky, "A phase balance relay of the negative phase sequence type", *Elect.J.*, vol.21, pp.77-81, 1924.

a.c. ammeter or relay in this branch will then be operated only by the negative sequence current.

The measurement of power in unsymmetrical systems. By an extension of the procedure just described it is clearly possible to devise networks which will be capable of isolating the positive, negative, or zero sequence currents, as desired. The currents so isolated can be passed into the current coils of suitable wattmeters or watt-hour meters, thus enabling the true power the positive, negative, or zero sequence volt-amperes to be measured. Fortescue,* Slepian, and other American engineers have largely developed this idea and have devised a range of instruments actuated on these principles.

* See C.L.Fortescue, "Polyphase power representation by means of symmetrical coordinates", *Journal Amer.I.E.E.*, vol.39, pp.543-544, 1920. Also other papers in the same volume.

Derivation of a single-phase supply from a polyphase network. An important technical problem, frequently occurring in connection with furnace work, is the supply of single-phase power from a polyphase circuit and the treatment of the resulting dissymmetry in the latter. This matter can easily be [illegible]e dealt with by the method of symmetrical coordinates.* The method also enables other cases of unsymmetrical supply to be treated simply, _e.g._, the supply for the Greaves-Etchells furnace.

*See R.E.Gilman and C.L.Fortescue, "Single-phase power service from central stations", _Proc.Amer.I.E.E._, vol.35, part II, pp.1431-1451, 1916. Similar problems are treated by Professor Miles Walker, "The supply of single-phase power from three-phase systems", _J.I.E.E._, vol.57, pp.109-139, 1919, by a method resembling that of symmetrical coordinates.

The air-gap field of a three-phase winding. The magnetic field in the air-gap of an induction motor or the armature reaction of an alternator has been investigated by many writers and the solution of the problem is very well-known for the case of a three-phase winding carrying a symmetrical system of currents. When the currents are unsymmetrical, the direct analysis becomes very complex; Dr. Clayton and, independently, the present author have both attempted solutions for unbalanced loads but the nature of the trigonometrical mathematics obscures the real physical simplicity of the problem. If the unsymmetrical system of currents be split up into its positive, negative, and zero sequence components the air-gap field is the sum of the fields produced by the components separately.

The positive sequence currents produce a wave of magnetic force of changing type sweeping round the armature periphery at synchronous speed in the same direction as the rotor or pole-wheel, the wave-length of the wave being a pole-pitch. The negative sequence currents produce a wave of magnetic force precise ly similar to that produced by the positive sequence currents but of a different amplitude and moving round the machine at synchronous speed in the opposite direction. The zero sequence or single-phase currents produce a wave of magnetic force which is stationary in space,of a wave-length equal to one-third of a pole-pitch,and which pulsates at synchronous frequency.

The forward,reverse,and stationary components of the armature field are easily treated by the well-known theory which has been worked out for symmetrical cases; The total field is then obtained by superposition of the component fields in their correct phase relationships as determined by these of the symmetrical coordinates of the unsymmetrical system of currents The analytical work is thus considerably simplified and the physics of the problem kept in view. Some aspects of the theory of the field produced by an armature winding carrying unsymmetrical currents will be treated by this method in a paper now in preparation by Mr.S.Neville and the author.

12. CONCLUSION.

In the preceding sections an attempt has been made to explain the principles underlying the vectorial treatment of polyphase theory and to apply those principles to unbalanced loads. It is the writer's hope that the explanations may assist students and others to use the vector methods in their work,and be of service to those who wish to read the original papers referred to in the text.

Finally, it remains for the author to record his indebtedness to his friends Messrs. F.Morley Colebrook and S.Neville for their valuable help and criticisms during the preparation of the paper, and to Professors G.W.O.Howe and S.Parker Smith for their encouragement and suggestions. The work was carried out at the James Watt Engineering Laboratories of the University of Glasgow.

DIAGRAMS.

FOOTLINES FOR DIAGRAMS.

Fig.1. Expression of a vector in terms of its rectangular components. The operator $\underline{j}$.

Fig.2. Relationship between two vectors $\underline{r} = r\epsilon^{j\theta}\underline{x}$ and $\underline{r}' = r'\epsilon^{j(\theta+\phi)}\underline{x} = \frac{r'}{r}\epsilon^{j\phi}\underline{r}$

Fig.3. Representation of a sinusoidally varying current by an harmonic vector.

Fig.4. Symmetrical polyphase systems of vectors in negative phase sequence, $1\underline{i}, \lambda\underline{i}, \lambda^2\underline{i}, \ldots \lambda^{(N-1)}\underline{i}$. $\underline{N}$ = number of phases, and λ = operator producing counterclockwise rotation of $2\pi/\underline{N}$.

Fig.5. Symmetrical polyphase systems of vectors in positive phase sequence, $1\underline{i}, \lambda^{N-1}\underline{i}, \lambda^{N-2}\underline{i}, \ldots \lambda\underline{i}$. $\underline{N}$ = number of phases, and λ = operator producing counterclockwise rotation of $2\pi/\underline{N}$.

Fig.6. Representation of a current harmonic of order $\underline{n}$ by a rotating vector; $\underline{i}_n$ = amplitude of the current.

Fig,7. Possible current harmonics in a three-phase circuit with independent phases. The diagrams for the possible voltages are presisely similar., λ = operator producing counterclockwise rotation of $2\pi/3 = \epsilon^{j2\pi/3}$.

Fig.8. Possible voltage harmonics in the phases and across the lines of a three-phase star-connected circuit. $\lambda = \epsilon^{j2\pi/3}$.

Fig.9. Showing the paths of the possible current harmonics in three-wire and four-wire three-phase circuits.

Fig.10. Possible current harmonics in the phases and lines of a three-phase mesh-connected circuit. $\lambda = \epsilon^{j2\pi/3}$.

Fig.11. (a) Possible voltage harmonics in the phases and across the lines of a three-phase mesh-connected circuit. (b) Three mesh-connected cells illustrating the properties of the 3$\underline{k}$th. harmonics in the three-phase mesh-connected circuit.

Fig.12. The resolution of an unsymmetrical three-phase system of currents $\underline{i}_{I}, \underline{i}_{II}, \underline{i}_{III}$, in a four-wire circuit into its symmetrical coordinates; and the superposition of the coordinates in the circuit. $\lambda = \epsilon^{j2\pi/3}$.

Fig.13. The resolution of anunsymmetrical three-phase system of currents $\underline{i}_{I}, \underline{i}_{II}, \underline{i}_{III}$, in a three-wire circuit into its symmetrical coordinates; and the superposition of the coordinates in the cdrcuit. $\lambda = \epsilon^{j2\pi/3}$.

Fig.14. Scalar relationships between an unsymmetrical three-phase system of vectors and its symmetrical coordinates.

Fig.15. Positive and negative phase sequence quarter-phase systems of currents.

Fig.16. The resolution of an unsymmetrical two-phase system of currents $\underline{i}_{I}$, $\underline{i}_{II}$, into (a) symmetrical coordinates, and (b) quarter-phase coordinates; and the superposititon of the current coordinates in a three-wire two-phase circuit.

Fig.17. The resolution of a quarter-phase system of currents $\underline{i}_{I}$, $\underline{i}_{II}$, into symmetrical coordinates.

Fig,18. Illustrating the production of power by an Nphase symmetrical system of voltages and the symmetrical coordinates of an unsymmetrical system of currents; (a) positive and negative sequence coordinates, and (b) zero sequence coordinates.

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Fig. 1.

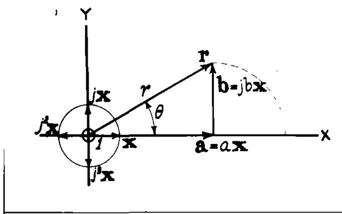


Fig. 2.

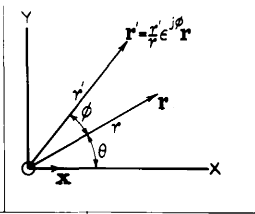
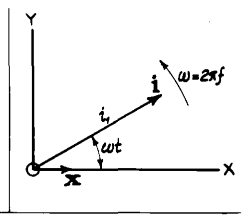


Fig. 3.



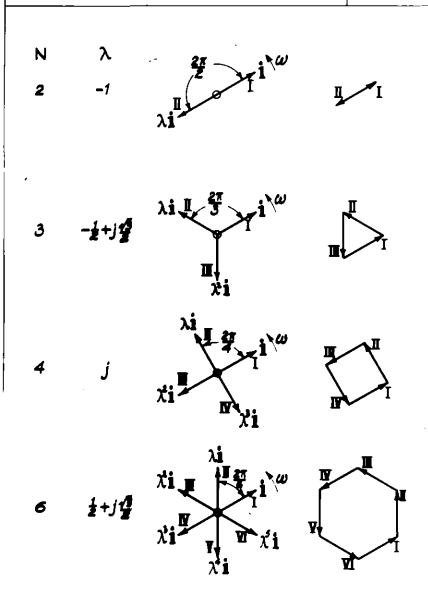


Fig. 4.

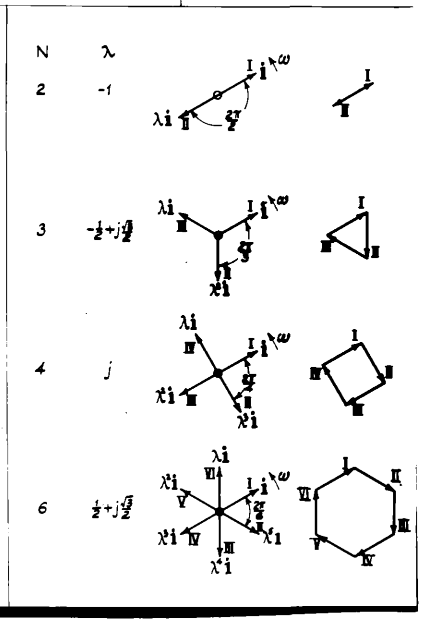
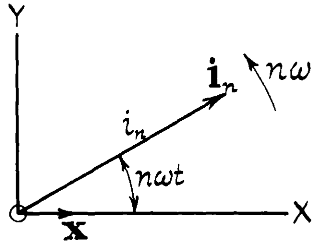
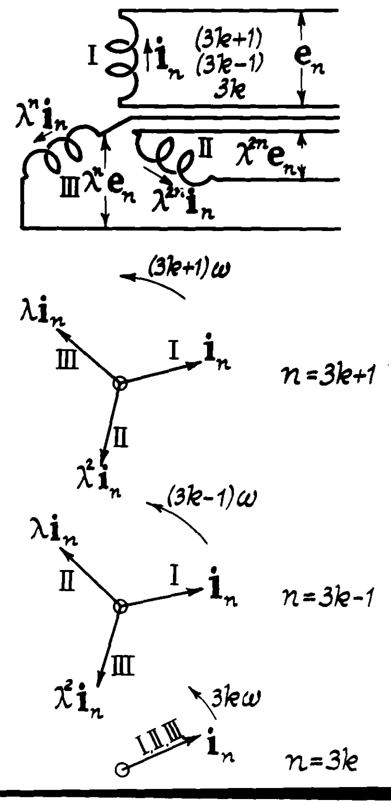


Fig. 5.

Fig. 6

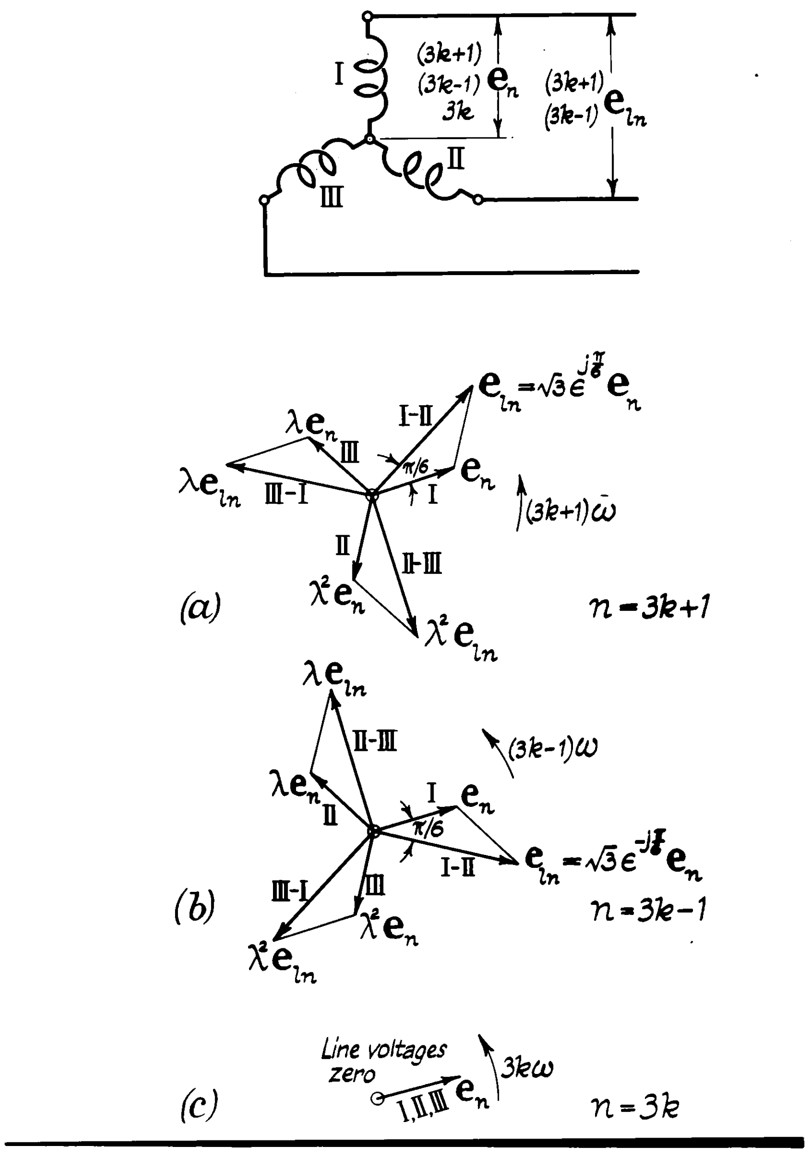


Y
$\mathbf{i}_n$
$n\omega$
i_n
$n\omega t$
X
$\mathbf{x}$



I
$\mathbf{i}_n$
(3k+1)
(3k−1)
3k
$\mathbf{e}_n$
$\lambda^n \mathbf{i}_n$
II
$\lambda^{2n} \mathbf{e}_n$
III
$\lambda^n \mathbf{e}_n$
$\lambda^{2n} \mathbf{i}_n$
(3k+1)ω
$\lambda \mathbf{i}_n$
III
I
$\mathbf{i}_n$
$n = 3k+1$
II
$\lambda^2 \mathbf{i}_n$
(3k−1)ω
$\lambda \mathbf{i}_n$
II
I
$\mathbf{i}_n$
$n = 3k-1$
III
$\lambda^2 \mathbf{i}_n$
3kω
I, II, III
$\mathbf{i}_n$
$n = 3k$

Fig. 7.



I
II
III
(3k+1)
(3k-1) $\mathbf{e}_n$
3k
(3k+1)
(3k-1) $\mathbf{e}_{ln}$
$\mathbf{e}_{ln} = \sqrt{3}\,\epsilon^{j\frac{\pi}{6}}\,\mathbf{e}_n$
$\lambda \mathbf{e}_n$
$\lambda \mathbf{e}_{ln}$
$\mathbf{e}_n$
I-II
III-I
II-III
$\pi/6$
$(3k+1)\omega$
$\lambda^2 \mathbf{e}_n$
$\lambda^2 \mathbf{e}_{ln}$
(a)
$n = 3k+1$
$\mathbf{e}_{ln} = \sqrt{3}\,\epsilon^{-j\frac{\pi}{6}}\,\mathbf{e}_n$
$(3k-1)\omega$
(b)
$n = 3k-1$
Line voltages zero
I, II, III
$3k\omega$
(c)
$n = 3k$

Fig. 8.

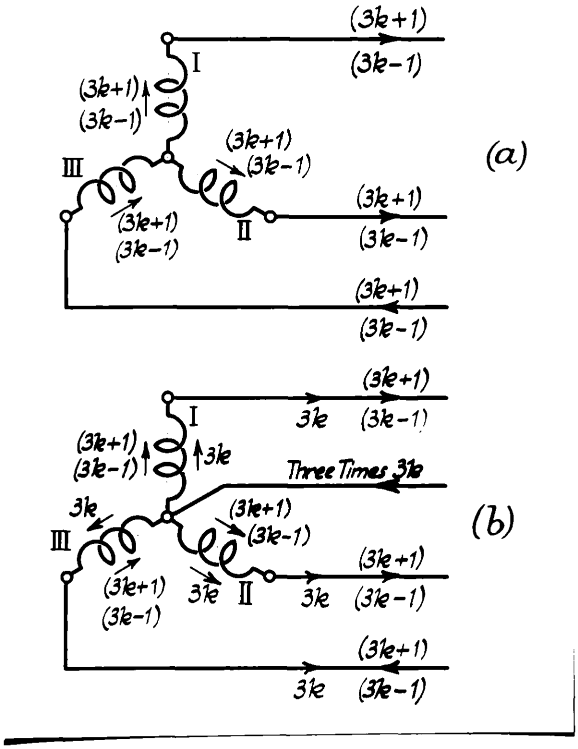
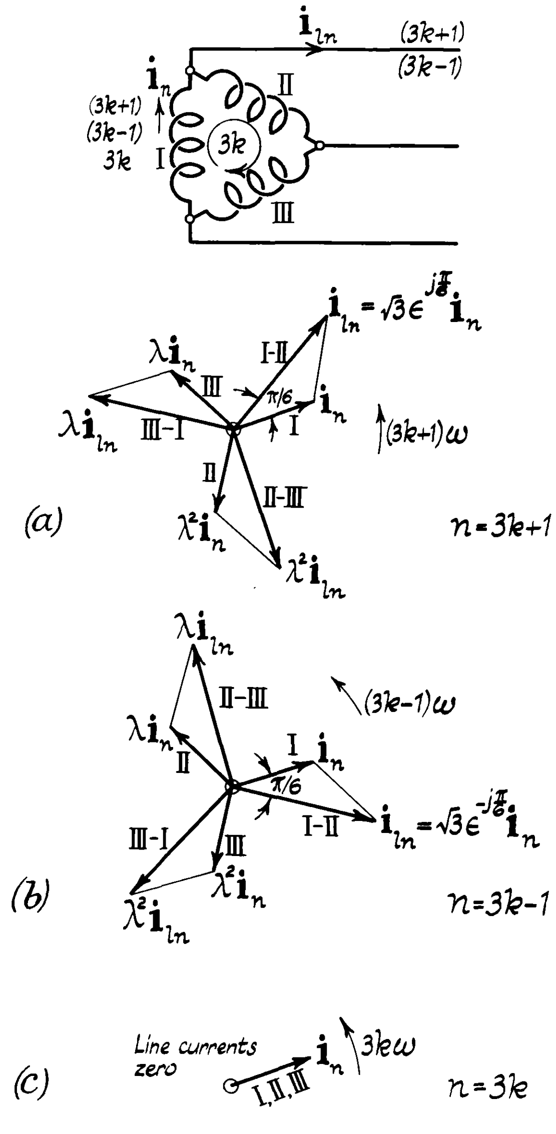


Fig 9.



$\mathbf{i}_{ln}$
(3k+1)
(3k−1)
$\mathbf{i}_n$
(3k+1)
(3k−1)
3k
I
II
III
3k
$\mathbf{i}_{ln} = \sqrt{3}\epsilon^{j\frac{\pi}{6}}\mathbf{i}_n$
$\lambda\mathbf{i}_n$
I-II
III
π/6
$\mathbf{i}_n$
$\lambda\mathbf{i}_{ln}$
III-I
I
(3k+1)ω
II
II-III
(a)
$\lambda^2\mathbf{i}_n$
n=3k+1
$\lambda^2\mathbf{i}_{ln}$
$\lambda\mathbf{i}_{ln}$
II-III
(3k−1)ω
$\lambda\mathbf{i}_n$
I
$\mathbf{i}_n$
II
π/6
III-I
I-II
$\mathbf{i}_{ln} = \sqrt{3}\epsilon^{-j\frac{\pi}{6}}\mathbf{i}_n$
III
(b)
$\lambda^2\mathbf{i}_n$
n=3k−1
$\lambda^2\mathbf{i}_{ln}$
Line currents zero
$\mathbf{i}_n$
3kω
(c)
I,II,III
n=3k

Fig. 10.

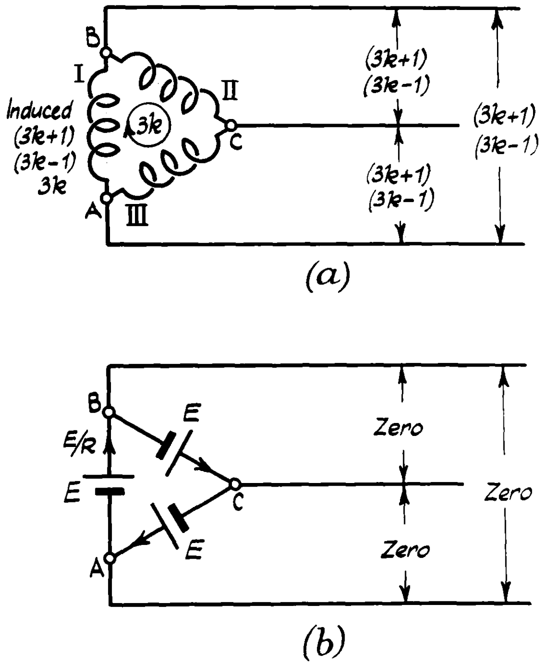

B
I
II
Induced
(3k+1)
(3k-1)
3k
3k
C
A
III
(3k+1)
(3k-1)
(3k+1)
(3k-1)
(3k+1)
(3k-1)
(a)
B
E
E/R
E
C
E
A
Zero
Zero
Zero
(b)

Fig 11

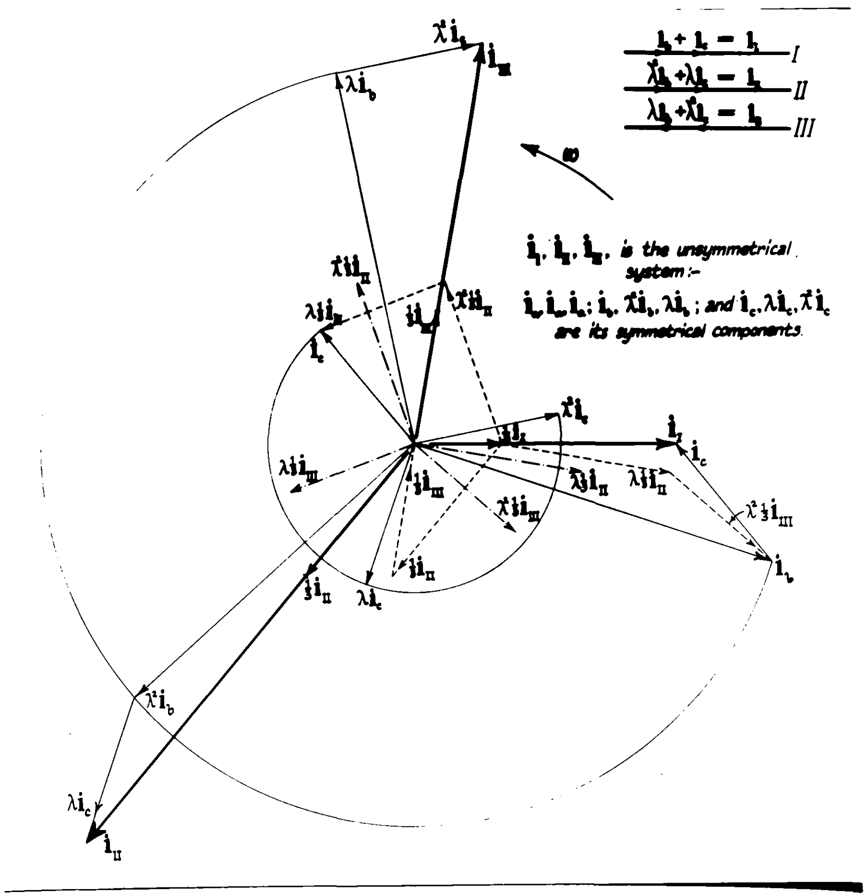
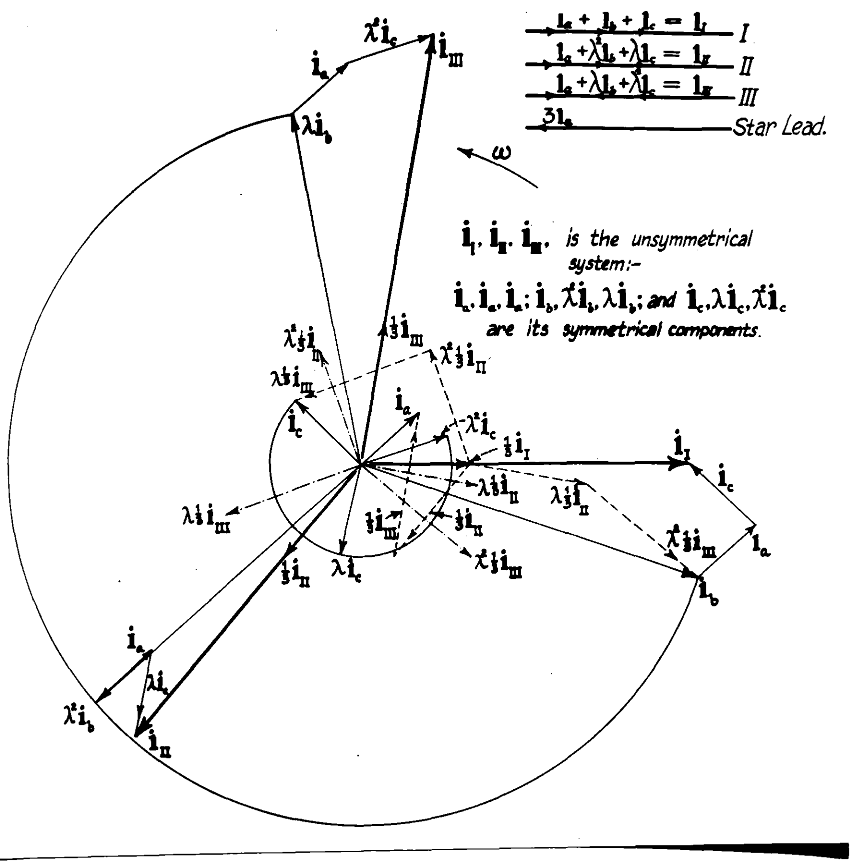

I
II
III
ω
is the unsymmetrical system:-
are its symmetrical components.

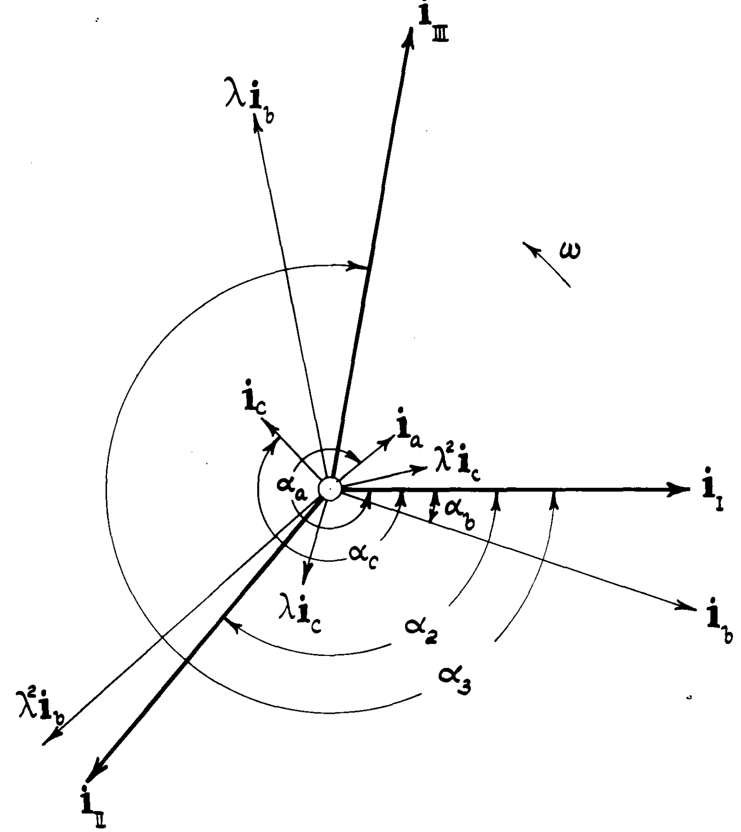
Fig 12



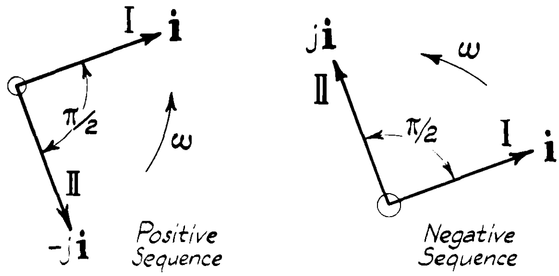
$i_a + i_b + i_c = i_I$ — I
$i_a + \lambda^2 i_b + \lambda i_c = i_{II}$ — II
$i_a + \lambda i_b + \lambda^2 i_c = i_{III}$ — III
$3i_a$ — Star Lead.
ω
i_I, i_{II}, i_{III}, is the unsymmetrical system:-
i_a, i_a, i_a; $i_b, \lambda^2 i_b, \lambda i_b$; and $i_c, \lambda i_c, \lambda^2 i_c$ are its symmetrical components.

Fig 13

Fig. 14.

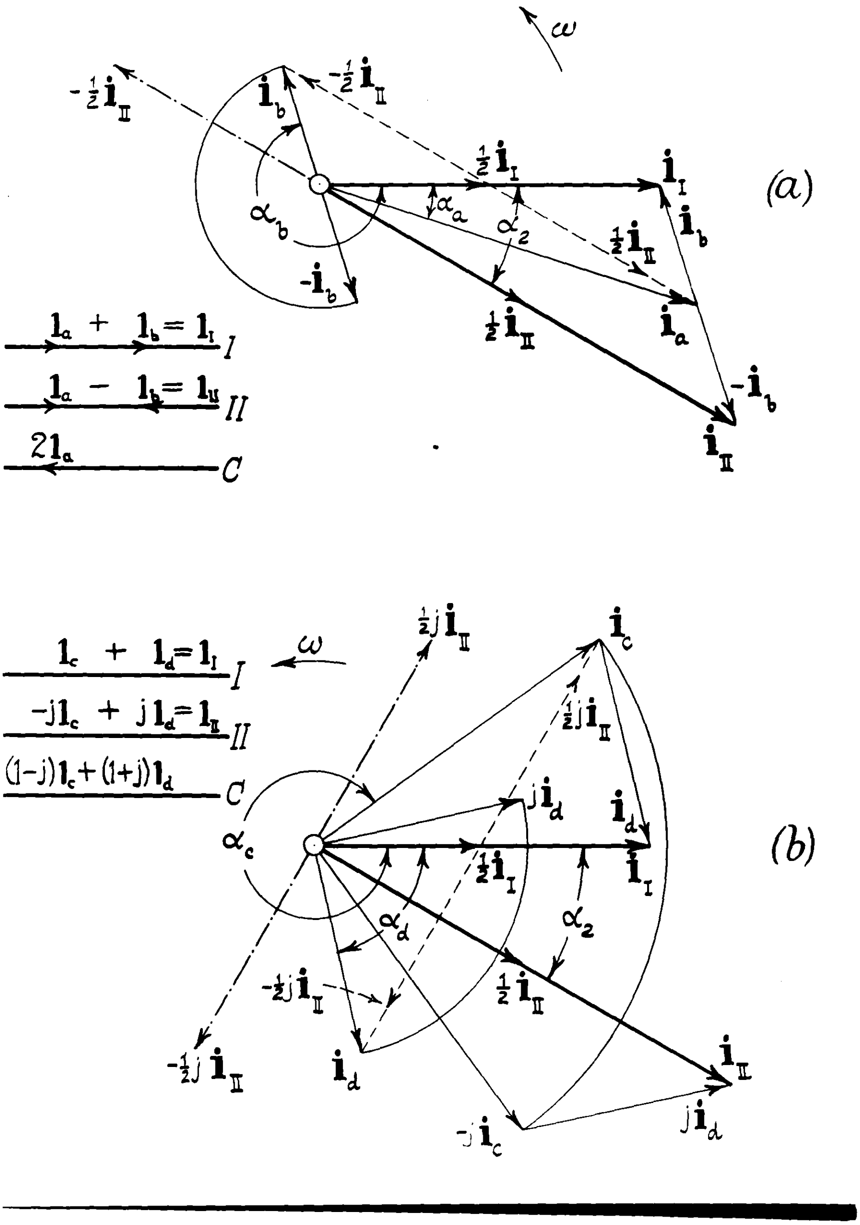


$\mathbf{i}_{III}$
$\lambda \mathbf{i}_b$
ω
$\mathbf{i}_c$
$\mathbf{i}_a$
$\lambda^2 \mathbf{i}_c$
α_a
$\mathbf{i}_I$
α_b
α_c
$\lambda \mathbf{i}_c$
$\mathbf{i}_b$
α_2
α_3
$\lambda^2 \mathbf{i}_b$
$\mathbf{i}_{II}$



I
$\mathbf{i}$
$\pi/2$
ω
II
$-j\mathbf{i}$
Positive Sequence
$j\mathbf{i}$
ω
II
$\pi/2$
I
$\mathbf{i}$
Negative Sequence

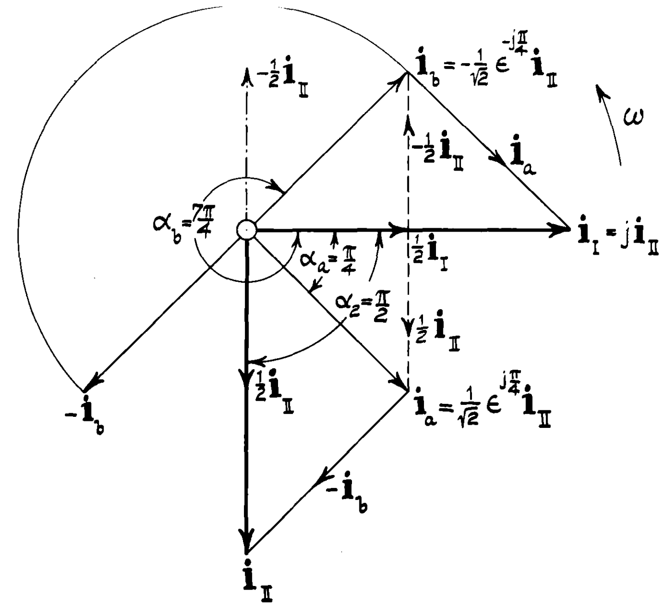
Fig. 15.



(a)
ω
Ia + Ib = II I
Ia − Ib = III II
2Ia C
(b)
ω
Ic + Id = II I
−jIc + jId = III II
(1−j)Ic + (1+j)Id C

Fig.16

Fig.17.


$\mathbf{i}_b = -\frac{1}{\sqrt{2}}\epsilon^{-j\frac{\pi}{4}}\mathbf{i}_{II}$
$-\frac{1}{2}\mathbf{i}_{II}$
ω
$\mathbf{i}_a$
$\alpha_b = \frac{7\pi}{4}$
$\alpha_a = \frac{\pi}{4}$
$\frac{1}{2}\mathbf{i}_I$
$\mathbf{i}_I = j\mathbf{i}_{II}$
$\alpha_2 = \frac{\pi}{2}$
$\frac{1}{2}\mathbf{i}_{II}$
$-\mathbf{i}_b$
$\mathbf{i}_a = \frac{1}{\sqrt{2}}\epsilon^{j\frac{\pi}{4}}\mathbf{i}_{II}$
$\mathbf{i}_{II}$

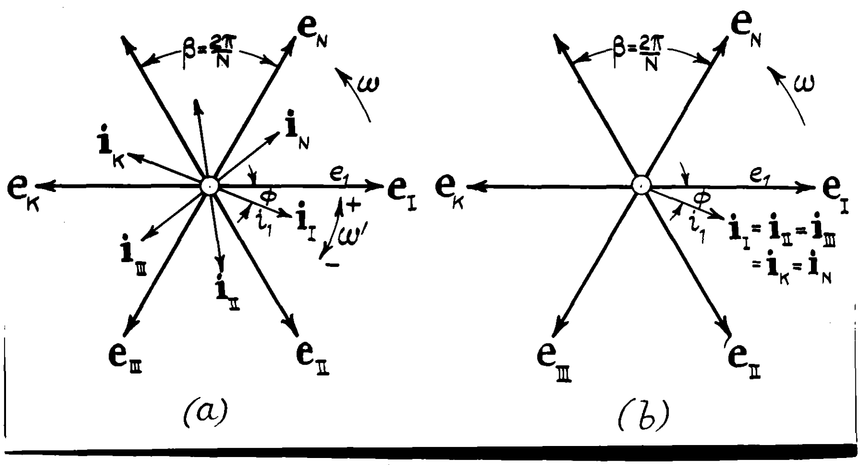

$\beta = \frac{2\pi}{N}$
$\mathbf{e}_N$
ω
$\mathbf{i}_N$
$\mathbf{i}_K$
$\mathbf{e}_K$
e_1
$\mathbf{e}_I$
ϕ
i_1
$\mathbf{i}_I$
ω'
$\mathbf{i}_{III}$
$\mathbf{i}_{II}$
$\mathbf{e}_{III}$
$\mathbf{e}_{II}$
(a)
$\mathbf{i}_I = \mathbf{i}_{II} = \mathbf{i}_{III} = \mathbf{i}_K = \mathbf{i}_N$
(b)

Fig.18.

THE DYNAMOMETER WATTMETER.

Some Notes on its Theory—The Application of Corrections.

By B. HAGUE, M.Sc. D.I.C., A.M I.E.E.

The object of this article is to draw attention to a detail in the theory of the dynamometer wattmeter which does not appear to be treated in text-books. It is well known that a wattmeter is subject to two principal sources of error: the first arises from the fact that the potential circuit of the instrument necessarily possesses some inductance; the second from the fact that the reading of the wattmeter includes not only the power which it is desired to measure, but also the electrical losses in the potential coil or in the current coil, according to the way in which the instrument is connected in circuit.

Figs. 1 and 2 show the two standard methods of joining up a wattmeter to measure the power passing from the alternating current supply at the left to a reactive load on the right of the diagrams.

Assuming at first that the potential coil of the wattmeter is joined to the alternator side of the current coil AB, as in Fig. 1, the reading of the instrument will in this case include

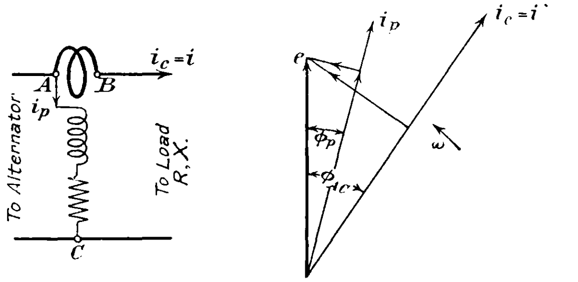


FIG. 1.

the power loss in the current coil. On the other hand, if the potential coil be connected to the load side of the current coil, as in Fig. 2, the instrument will measure the loss in the potential circuit in addition to the power supplied to the load. Moreover, in both these cases, the readings will be affected by the presence of reactance in the potential circuit.

The point which it is the object of this article to make clear, and which is not treated in text-books, is that the order in which the two corrections: (i) for the instrument losses and (ii) for the potential coil reactance, are to be applied to the instrument reading is different in the two cases mentioned above.

Let the wattmeter be of the zero pattern, in which the plane of the potential coil is maintained at right angles to that of the current coil by the application of a torque from a spring and torsion head. The torque at any instant acting on the potential coil is proportional to the product of the current i_p in it and the current i_c in the fixed current coils. If the currents are alternating with a period T seconds, the average torque on the potential coil tending to deflect it will be

$$\frac{a}{T}\int_o^T i_p i_c dt,$$

where a is a constant depending on the numbers of turns in the coils and on their relative proportions. If to maintain the coils at right angles it is necessary to rotate the torsion head through an angle θ radians the torque on the spring will be $c\theta$, c being the torque per radian twist; then

$$c\theta = \frac{a}{T}\int_o^T i_p i_c dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is the fundamental equation for the action of a dynamometer instrument. The two cases can now be considered, in order to evaluate i_p and i_c.

Case I.

Consider the arrangement shown in Fig. 1, and let the voltage e across the potential circuit, *i.e.*, across the points AC be

$$e = e_1 \sin \omega t,$$

where e_1 is the maximum value of the voltage and $\omega = 2\pi/T$. Then if R_p be the resistance and L_p the inductance of the potential circuit

$$i_p = \frac{e_1}{Z_p}\sin(\omega t - \varphi_p) = \frac{e_1}{R_p}\cos\varphi_p . \sin(\omega t - \varphi_p),$$

where

$$Z_p = \sqrt{R^2_p + \omega^2 L^2_p} \text{ and } \varphi_p = \tan^{-1}(\omega L_p / R_p).$$

The current in the current coil is the same as that in the load. If R, X be the equivalent resistance and reactance of the load, R_c, L_c the resistance and inductance of the current coil AB then

$$i_c = i = \frac{e_1}{Z_{AC}}\sin(\omega t - \varphi_{AC}) = i_1 \sin(\omega t - \varphi_{AC}),$$

Where

$$Z_{AC} = \sqrt{(R+R_c)^2 + (X+\omega L_c)^2}, \text{ and } \varphi_{AC} = \tan^{-1}\frac{X+\omega L_c}{R+R_c}.*$$

Substituting in equation 1, the average torque is

$$c\theta = \frac{a}{T}\int_o^T \frac{e_1 i_1}{R_p}\cos\varphi_p \sin(\omega t - \varphi_p)\sin(\omega t - \varphi_{AC})\,dt$$

$$= \frac{a e_1 i_1}{2R_p}\cos\varphi_p \cos(\varphi_{AC} - \varphi_p).$$

If E, I be the r.m.s. values of e and i, this result becomes

$$\frac{c}{a}R_p\theta = EI\cos\varphi_p\cos(\varphi_{AC} - \varphi_p).$$

The right hand side of this equation has the dimensions of power; hence the factor $\frac{c}{a}R_p$ converts the angular reading of the torsion head into power units. The left-hand side is the power indicated by the wattmeter, P_w say. Inserting this symbol, multiplying both sides by $\cos\varphi_{AC}$ and transposing gives

$$\frac{\cos\varphi_{AC}}{\cos\varphi_p\cos(\varphi_{AC}-\varphi_p)}P_w = EI\cos\varphi_{AC}$$

$$= \frac{EI}{Z_{AC}}(R_c + R)$$

$$= I^2 R_c + I^2 R.$$

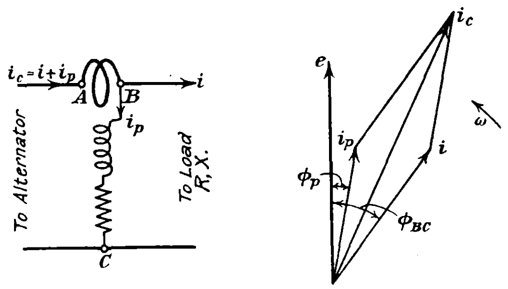


FIG. 2.

Now I^2R is the power expended in the load, I^2R_c the loss in the current coil; hence if the correcting factor is

$$K_{AC} = \frac{\cos\varphi_{AC}}{\cos\varphi_p\cos(\varphi_{AC}-\varphi_p)},$$

Power in load $= K_{AC}$ *(Wattmeter Reading)—Power loss in current coil.*

Case II.

Let the wattmeter be connected as in Fig. 2 and again let the voltage across the potential points, in this case B and C, be

$$e = e_1 \sin \omega t;$$

then as before

$$i_p = \frac{e_1}{R_p}\cos\varphi_p \sin(\omega t - \varphi_p).$$

The current through the load will be

$$i = i_1 \sin(\omega t - \varphi_{BC}),$$

where $\quad i_1 = \dfrac{e_1}{\sqrt{R^2+X^2}}$ and $\varphi_{BC} = \tan^{-1}\dfrac{X}{R}.$

* When the load is a simple conducting circuit R and X are its true resistance and reactance. If, however, the load be composed of some piece of apparatus such as a transformer, then R and X are to be taken as the resistance and reactance of a conducting network equivalent thereto.

The current in the current coil will be

$$i_c = i_p + i = \frac{e_1}{R_p} \cos \varphi_p \sin (\omega t - \varphi_p) + i_1 \sin (\omega t - \varphi_{BC}).$$

Inserting these values in equation.

$$c0 = \frac{a}{T} \int_o^T \left\{ \frac{e_1}{R_p} \cos \varphi_p \sin (\omega t - \varphi_p) \left[\frac{e_1}{R_p} \cos \varphi_p \sin (\omega t - \varphi_p) + i_1 \sin (\omega t - \varphi_{BC}) \right] \right\} dt$$

$$= a \left[\frac{E^2}{R^2_p} \cos^2 \varphi_p + \frac{EI}{R_p} \cos \varphi_p \cos (\varphi_{BC} - \varphi_p) \right],$$

making use of the r.m.s. values E and I of e and i.

Converting the scale reading to power units gives

$$P_w = \frac{E^2}{R_p} \cos^2 \varphi_p + EI \cos \varphi_p \cos (\varphi_{BC} - \varphi_p)$$

$$= R_p \frac{E^2}{Z^2_p} + EI \cos \varphi_p \cos (\varphi_{BC} - \varphi_p)$$

$$= I^2_p R_p + EI \cos \varphi_p \cos (\varphi_{BC} - \varphi_p),$$

where I_p is the r.m.s. value of i_p. Multiplying by $\cos \varphi_{BC}$ and transposing

$$(P_w - I^2_p R_p) \frac{\cos \varphi_{BC}}{\cos \varphi_p \cos (\varphi_{BC} - \varphi_p)} = EI \cos \varphi_{BC}.$$

Now $EI \cos \varphi_{BC}$ is the power expended in the load, while $I^2_p R_p$ is the potential coil loss; writing the correcting factor as

$$K_{BC} = \frac{\cos \varphi_{BC}}{\cos \varphi_p \cos (\varphi_{BC} - \varphi_p)};$$

Power in load $= K_{BC}$ (*Wattmeter reading—Power loss in potential coil*).

Conclusion.

On comparing Cases I and II two interesting facts emerge. In Case I the correction for the reactance of the potential circuit is applicable to the wattmeter reading *before* the correction for the instrument loss. In Case II precisely the reverse holds, the equation indicating that the reactance correction is to be applied *after* the wattmeter reading has been reduced by the amount of the instrument loss. Again, it will be observed that although the correcting factors are in each case of the same form, in Case II the phase-angle of the load alone is involved, whereas in Case I the angle is the total phase-displacement over the load and the current coil in series. In general, the difference between φ_{AC} and φ_{BC} is slight, since R_c and ωL_c are small quantities and are negligible in comparison with R and X. The difference may be of importance, however, when R and X are also small.

PREFACE

THE object of the author in preparing the present volume is to deal with the subject of Alternating Current Bridge Measurements of Inductance, Capacitance, and Effective Resistance at low and telephonic frequencies in a manner suited to the needs of the advanced student. The importance of such measurements in modern laboratory and test-room practice, in research work and in the training of students, would seem to be sufficient reason for the publication of a handbook dealing fairly completely with all the matters involved. As the book is intended primarily for practical use, every endeavour has been made to make clear the experimental side of the subject. At the same time an attempt has been made to provide a logical treatment of the theory underlying the use of a.c. bridge networks, since this is a matter which falls outside the scope of text-books dealing with the theory of alternating currents.

The book is based on a course of lectures given for the past three years to third-year students of the City and Guilds (Engineering) College, amplified by the addition of material intended to make the volume useful to post-graduate workers and to others engaged on original research or accurate testing. The subject-matter is divided into five chapters, each dealing with some aspect of the theme. The object of Chapter I is to define the various quantities which are to be dealt with in the rest of the book; considerable attention has been paid to the discussion of electrostatic phenomena. In Chapter II the theory of alternating currents is developed from the standpoint of the symbolic method, and an attempt is made to show the true relationship between the symbolic method and the more usual vector diagram and mathematical treatment of a.c. problems.

The apparatus required for bridge measurements is considered at some length in Chapter III, attention being chiefly directed to the explanation of the principles underlying the action of the various instruments rather than to a catalogue-like description of constructional details. The various bridge networks are classified in Chapter IV, the theory, uses, and full details of laboratory procedure being given in each case. With few exceptions, typical measurements have been made by all the methods described in this chapter, the results being included in the text as a guide to the student in carrying out his own experiments. Finally, the choice of the method suitable for a given measurement and the general precautions to be observed in laboratory practice are dealt with in Chapter V.

In preparing the book, full advantage has been taken of the information contained in original papers widely scattered throughout technical literature, complete references being given in the footnotes. In particular, the writings of Mr. A. Campbell and his associates at the National Physical Laboratory, of the papers published by the Washington Bureau of Standards, and by the Physikalische Technische Reichsanstalt have been drawn upon to a considerable extent. The manuscript of the book was in the hands of the printer before Mr. Campbell's informative articles in the *Dictionary of Applied Physics* were published, but reference has been made to them in revising the proofs for press.

In conclusion, thanks are due to many friends for help, advice and criticism during the preparation of the manuscript and proofs. In particular, the author wishes especially to thank Professor T. Mather, F.R.S., who not only suggested the preparation of the book, but who gave his unstinted help throughout. He has read the entire text in proof, and has provided the foreword. Mr. G. W. Sutton, B.Sc., has helped with the revision of the final proofs. Mr. S. Butterworth, M.Sc., of the Admiralty Research Laboratory, has kindly read the sections of the book on which he is an acknowledged authority.

The diagrams were all specially prepared for the book by the author with the skilled assistance of Mr. M. G. Say, M.Sc., to whom thanks are accorded. The author also wishes to record his indebtedness to the various firms, at home and abroad, who have kindly supplied information during the preparation of Chapter III.

LONDON.
July, 1923.

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CHAPTER IV

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CHAPTER V

ON THE CHOICE OF A BRIDGE METHOD AND THE PRECAUTIONS TO BE OBSERVED WHEN USING IT

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PRINTED IN GREAT BRITAIN AT THE PITMAN PRESS, BATH
(4435W)